Test-case number 1: Rise of a spherical cap bubble in a stagnant liquid (PN)

10 April 2003

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1 Practical significance and interest of the test-case

This test-case could usually be considered as a very preliminary one for a new numerical method. An extensive tester may want to reproduce most parts of the Clift, Grace and Weber map (Clift *et al.* , 1978). However, this selected case deserves special attention for the result not only consists in a final shape of the bubble (that is nevertheless a real criteria of comparison) but also in a precise build-up of the bubble velocity, starting from rest, exhibiting an overshoot before reaching its final asymptotic value.

To get the proper results, mainly the correct terminal velocity, and to reproduce the overshoot, a numerical method has to take accurately into account buoyancy, viscous stresses and surface tension effects. In particular, this test allows validating the numerical model that takes care of jump conditions at the interface (see *e.g.* (Scardovelli & Zaleski, 1999)). However, the test is less severe than the "Free rise of a liquid inclusion in a stagnant liquid", a test-case proposed by Lemonnier and Hervieu, presented in this volume.

2 Definitions and model description

The situation of the test-case is relative to a fluid inclusion rising in another fluid. The inclusion and the surrounding fluid are initially at rest. Gravity induced buoyancy is the only force inducing the motion. The test-case consists in the computation of the transient build-up of the velocity of the rising inclusion that finally reaches a constant value.

The physical model is reduced to Navier-Stokes equation in both phases, a constant surface tension at the interface. No phase-change takes place at the interface. As the solution does not depend on a possible compressibility of one or both of the phases, the test-case can be conducted in both cases (compressible / incompressible), depending on specific features of the numerical method to be evaluated.

Reference calculations are available in non-dimensional units; however, a typical set of dimensional physical parameter is suggested. The length scale of the problem is the initial diameter d_e of the inclusion. The velocity scale for speed of displacement of the center of mass U is,

$$U_c = \sqrt{g d_e}, \tag{1}$$

where g is the acceleration of the gravity. The time scale is therefore

$$t_c = \sqrt{d_e/g}. \tag{2}$$

Reduced parameters are $\tau = t/t_c$ and $u = U/U_c$. According to these definitions, the non-dimensional reference calculation consist in the reduced time evolution of the speed of displacement of the center of mass:

$$u = U/U_c = f(\tau) = f(t/t_c). \tag{3}$$

The computation can be conducted either for an axisymmetrical domain or in a true three-dimensional domain. As the limited extend of the domain has an impact on the terminal velocity of the inclusion (see *e.g.* Harmathy (1960)), the size of the computational domain has to be increased as long as an effect on the results is noted. As a rough first estimate, we suggest that that the computational domain has a minimal extend equal to ten diameters in all directions. According to the work of Harmathy (1960), the shape of the bubble is not affected by the domain extension whereas the terminal rising velocity modification can be estimated through the semi-empirical relation

$$\frac{U_c^{confined}}{U_c^{\infty}} \approx 1 - \left(\frac{d_e}{D}\right)^2, \tag{4}$$

where D is a characteristic dimension of the domain in a plan perpendicular to the gravity direction.

The physical parameters, namely ρ_L and ρ_V, respectively the density of the surrounding fluid and the fluid of the inclusion, μ_L and μ_V the dynamic viscosities and σ the surface tension, are chosen to get proper values of the non-dimensional quantities for which reference computations are available: the Morton number Mo, the Bond number Bo and the ratio of densities ρ_L/ρ_V and viscosities μ_L/μ_V. The Morton number and the Bond number are defined as usually by,

$$Mo = \frac{g\ \mu_L^4}{\rho_L\ \sigma^3}, \tag{5}$$

and

$$Bo = \frac{\rho_L\ g\ d_e^2}{\sigma}. \tag{6}$$

3 Summary of the requested calculations

- Compute the displacement of an inclusion with the following non-dimensional properties $Mo = 0.056$, $Bo = 40$, $\rho_L/\rho_V = 100$ and $\mu_L/\mu_V = 100$.
- As an example, we suggest the following physical properties: $\rho_L = 1000\ kg.m^{-3}$, $\rho_V = 10\ kg.m^{-3}$, $\mu_L = 0.273556\ Pa.s^{-1}$, $\mu_V = 0.00273556\ Pa.s^{-1}$, $\sigma = 0.1\ N.m$, $g = 10\ kg.s^{-2}$, $d_e = 0.02\ m$.
- Extract the position of the center of mass of the inclusion and then deduce its speed of displacement. The first point of comparison is the value of the reduced asymptotic velocity. This value can be obtained even with a peculiar point of the interface, such as the apex. Of course, in this later case, the temporal evolution around the overshoot (Figure 1) cannot be capture.

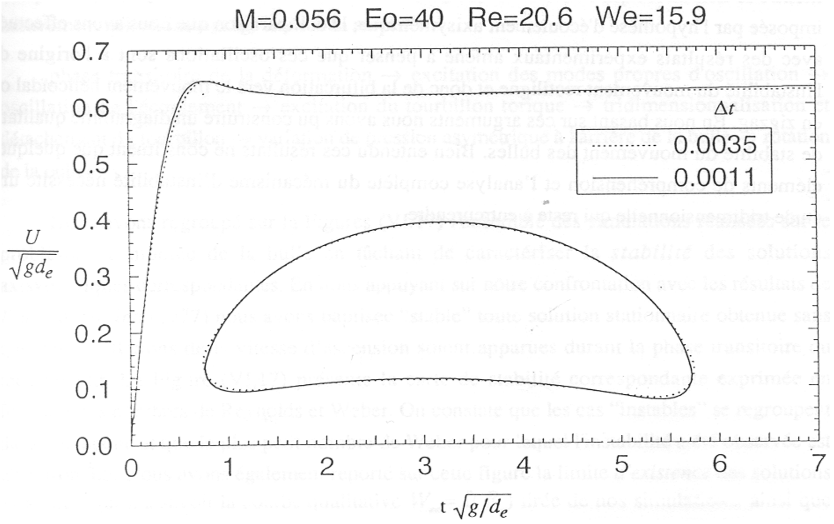

Figure 1: Reduced time evolution of the speed of displacement for two mesh sizes. After a figure of Blanco-Alvarez (1995).

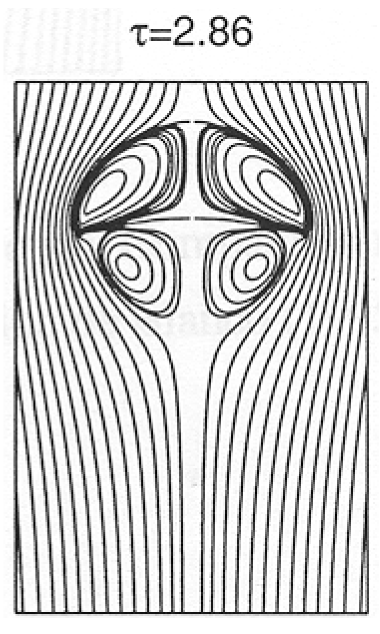

Figure 2: Recirculation zone and steam lines at reduced time $\tau = 2.86$. After a figure of Benkenida (1999).

- In addition to the main result, additional features consist in comparisons of the non-dimensional values of the over-shoot in the build-up of velocity (Figure 1).
- Further comparisons are the shape of lines of current, the equilibrium shape of the inclusion and the size of the recirculation zone (Figure 2). This late characteristic requires that the inclusion has risen a length of more than ten diameters (Hnat & Buckmaster, 1976).

References

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