Test-case number 10: Parasitic currents induced by surface tension (PC)

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1 Practical significance and interest of the test-case

The assessment of the consistency of a numerical model is proposed by comparison to theoretical results. The surface tension modeling is tested thanks to two analytical test cases. The first problem consists in verifying the equilibrium of a cylindrical drop initially at rest. The pressure in the drop is defined analytically (Laplace law). In the second test, an initially square cylindrical drop is oscillating under surface tension forces. The viscous damping of drop oscillations around a cylindrical shape is studied in order to reach a steady cylindrical final state corresponding to the Laplace problem. The oscillation frequency is known theoretically. The aim of the the two cases is to estimate the discretization error on the curvature of a free surface leading to the generation of parasitic numerical currents.

- in the first test 10a, the non-conforming of the numerical free-surface shape with the theoretical one induces local pressure variations in the drop. These pressure gradients lead to local velocities and to the propagation of surface waves of small amplitude. This spurious behavior is refereed to as parasitic currents.

- in the second test 10b, the oscillation frequency is well recovered by numerical methods with almost several percent error. However, the steady cylindrical state exhibits parisitc currents as in the previous test.

2 Definitions and physical model description

Two-dimensional configurations are considered where a viscous liquid drop is initially centered in a square cavity of characteristic length L, full of air. In test 10a, the radius of the circular shape drop is R_0 whereas l is the typical length scale of the square drop of test 10b. A zero-gravity field is imposed, the flow is assumed isothermal and the surface tension coefficient is constant. The tests can be easily extended to three dimensions.

With respect to the length scale of the problems, analytical results can be obtained on the pressure and oscillation frequency:

- for test10.a, the droplet keeps a cylindrical (or spherical) shape during time such that $R(t) = R_0$ and the pressure jump between the inside and outside drop pressures p_l and p_g is given by the Laplace equation

$$\begin{aligned} p_l - p_g &= \frac{2\sigma}{R_0}\text{(3D case)} \\ p_l - p_g &= \frac{\sigma}{R_0}\text{(2D case)} \end{aligned} \tag{1}$$

- for test 10.b, the initial square drop shape corresponds to a mode $n = 2$ perturbation. We compare the initial square drop to a cylindrical drop of same volume, radius of which is defined by

$$R_1 = \left(\frac{3}{4\pi}\right)^{1/3} l(3\text{D case})$$
$$R_1 = \pi^{-1/2} l(2\text{D case}) \tag{2}$$

The oscillation period T_0 can be estimated by

$$T_0 = \frac{2\pi}{\omega_0} \tag{3}$$

where

$$\omega_0 = (n^3 - n)\frac{\sigma}{(\rho_l + \rho_g)R_1^3}. \tag{4}$$

This period remains constant during time. Only the magnitude of the oscillations are diminishing under viscous effects to converge to a circular (or spherical) shape of radius R_1.

Numerical methods for front tracking or interface capturing are demonstrated to generate artificial numerical flows instead of keeping steady cylindrical drops (or spherical shapes in 3D). Following the work of Lafaurie *et al.* (1994), the order of magnitude of the spurious velocities u_p can be estimated according to the surface tension coefficient σ and dynamic viscosity μ of the drop,

$$u_p = C_p \frac{\sigma}{\mu} \tag{5}$$

where C_p is a numerical constant characteristic of the quality of the numerical modeling of surface tension forces (a non-dimensional number similar to a capillary number). The optimal value of C_p is zero. Typical values of C_p are found between 10^{-3} and 10^{-10}.

3 Test-case description

The fluid characteristics are $\rho_l = 797.88$ kg.m^{-3} and $\mu_l = 1.2\ 10^{-3}$ Pa.s for ethanol and $\rho_g = 1.1768$ kg.m^{-3} and $\mu_g = 10^{-5}$ Pa.s for air. The surface tension coefficient between ethanol and air is $\sigma = 0.02361$ N.m^{-1}. Initially, the velocity field is zero in the whole domain. Wall boundary conditions are considered in the two problems. The parasitic currents are observed in numerical simulations whatever the grid type and size. For test 10.a, the geometrical parameters are

- $R_0 = 210^{-3}$ m
- $L = 7.510^{-3}$ m

whereas for test 10.b, we choose

- $l = 410^{-2}$ m
- $L = 7.510^{-2}$ m

R_0/L	0.1	0.175	0.25	0.325	0.4
$\frac{\Delta p R}{\sigma}$	0.95	1.015	1.02	1.01	0.985
Theoretical value	1	1	1	1	1

R_0/Δ_x	3	6	10	13	16	20
$\frac{\Delta p R}{\sigma}$	0.925	0.98	1.02	1.01	0.99	0.995
Theoretical value	1	1	1	1	1	1

Table 1: Two-dimensional VOF-PLIC simulation of Laplace equation. Convergence test according to drop radius (top) and mesh size (bottom)

4 Example of comparison exercise

The Navier-Stokes equations in their single-fluid formulation for multiphase flows presented by Vincent & Caltagirone (2000) are implemented on a fixed cartesian grids for the two proposed problems. A Piecewise Linear Interface Construction of Youngs (1982) associated to a Volume Of Fluid (VOF) function C is used to track the free surface and the surface tensions are modeled thanks to the Continuum Surface Force (CSF) of Brackbill *et al.* (1992). In addition, test case 10.a is computed with an Eulerian-Lagrangian front-tracking method of Shin & Juric (2002) with the surface tension forces modeled by using the Fresnet relation and Peskin approximation.

The table 1 and figure 1 relative to test 10.a show that the VOF-PLIC approach induces the generation of spurious currents which disturb the convergence of the solution towards the known equilibrium. On the contrary, the front tracking method gives a better approximation of the Laplace equation on the same grid and reduces the velocities in the drop. For the case presented on figure 1, $C_p = 5 \cdot 10^{-5}$ for the VOF-PLIC method and $C_p = 5 \cdot 10^{-7}$ for the front tracking method, corresponding to $u_p = 10^{-3}$ m.s^{-1} and $u_p = 10^{-5}$ m.s^{-1} respectively inside ethanol. The VOF-PLIC numerical simulation of problem 10.b is presented on figure 2. The 0.4s period of drop oscillation presented by Brackbill *et al.* (1992) is recovered. As in the previous reference simulations, spurious currents are observed for long calculation times which prevent the drop from converging to a cylindrical shape under viscous effects.

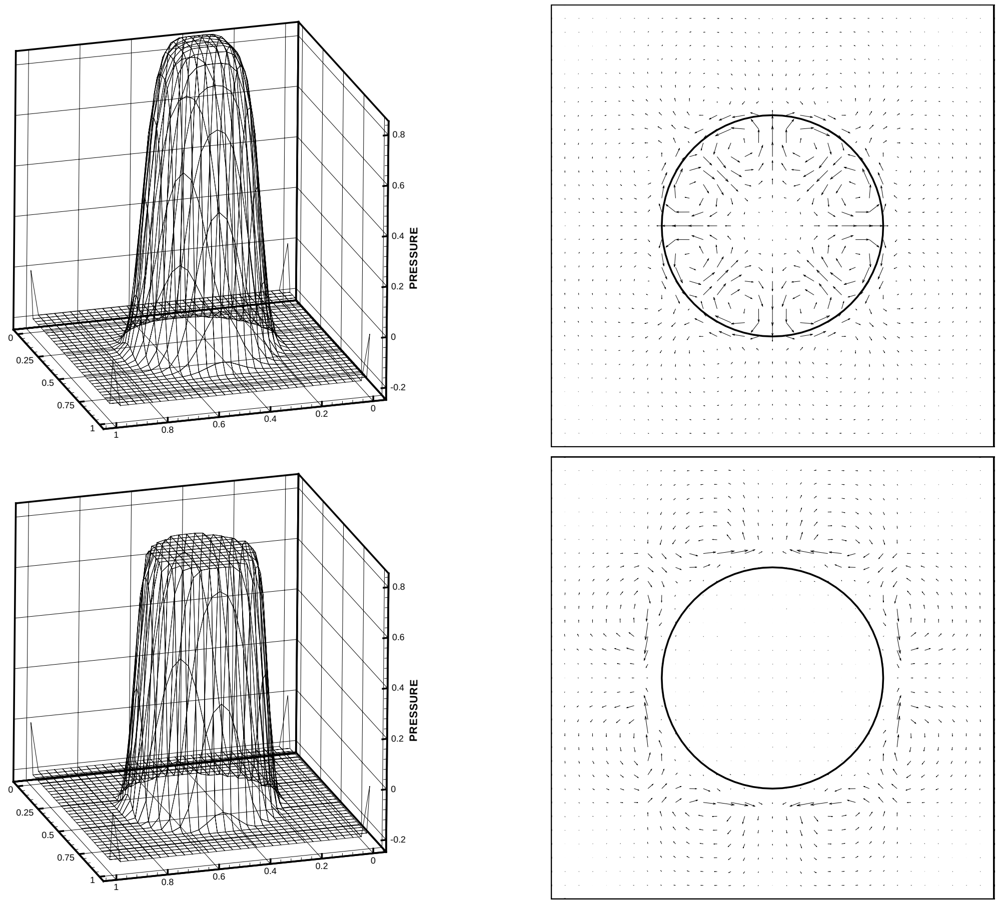

Figure 1: Two-dimensional simulation of Laplace equation with VOF-PLIC (top) and front-tracking (bottom) methods on a 30 x 30 grid. The pressure is plotted on the left whereas the velocity field and the free surface are shown on the right. The reference pressure jump across interface given by equation (1) is 1 Pa.

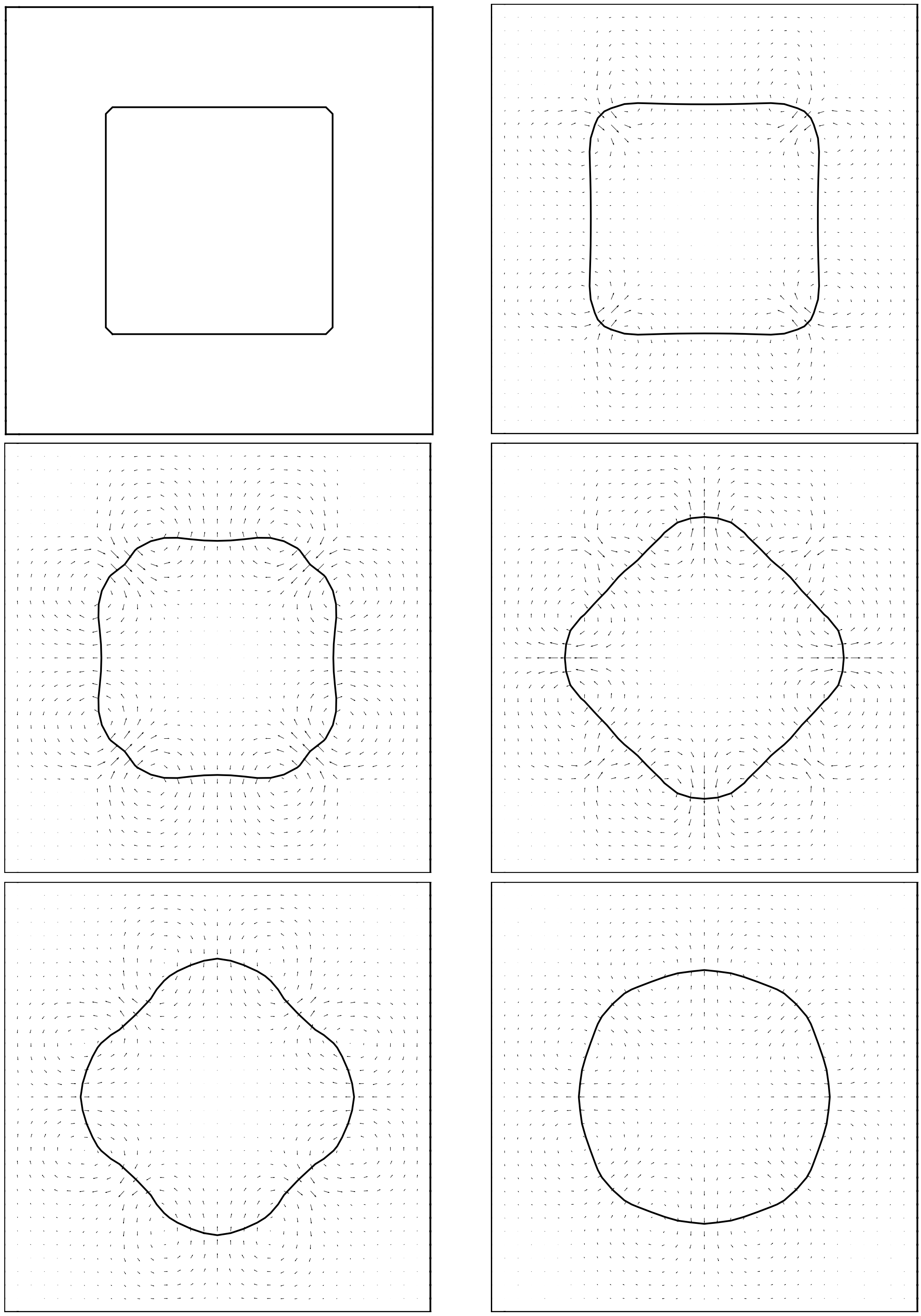

Figure 2: Simulation of square drop oscillation under surface tension force with VOF-PLIC and CSF methods on a 30 x 30 grid. The velocity field and the free surface are plotted for $t = 0$, 0.05, 0.1, 0.2, 1 et 5 s (from left to right and from top to bottom). Figure at time $t = 5$ s emphasizes the presence of spurious currents in a near equilibrium drop state.

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