Test-case number 11a: Translation and rotation of a concentration disk (N)

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1 Practical significance and interest of the test-case

The aim of the test is to validate the advection algorithm for volume fraction for analytical velocity field involving only translation and rotation. In this way, no deformations of the initial interface shape are induced. The interest of advection problems is that they can be handled by all numerical interface tracking methods, whatever their type Lagrangian or Eulerian, with a low computer cost. Two configurations are proposed concerning the advection of a circular concentration field in a single vortex (case 11a.a) and the rotation of a hollow disk in the same velocity field. The numerical diffusion, mass conservation and advection accuracy can be estimated to demonstrate the overall quality of several interface tracking techniques. More details are available in the work of Unverdi & Tryggvason (1992) and Rudman (1997).

2 Definitions and physical model description

In two-phase flow modeling, the volume fraction or concentration C is a local function defined between 0 and 1 which is characteristic of the presence of one of the two fluids. The interface between the two phases is defined as $C = 0.5$. In an analytical velocity field $\mathbf{u}$, the advection of the volume fraction is solved through a passive scalar equation

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = 0 \tag{1}$$

where t denotes time.

3 Test-case description

In a square box with a side lenght of 1 meter, a concentration zone is initialized as follows:

- Case 11a.a: A cylindrical interface is placed at point $(0.25, 0.5)$ with radius $R_0 = 0.15$ m.
- Case 11a.b: A cylindrical interface is placed at point $(0.5, 0.5)$ with radius $R_0 = 0.25$ m. This disc is cut down by a rectangle whose two opposite corners are $(0.5, 0.45)$ and $(0.75, 0.55)$.

The same velocity field is used for the two test cases:

$$\mathbf{u}(x, y) = \begin{pmatrix} 0.5 - y \\ x - 0.5 \end{pmatrix} \tag{2}$$

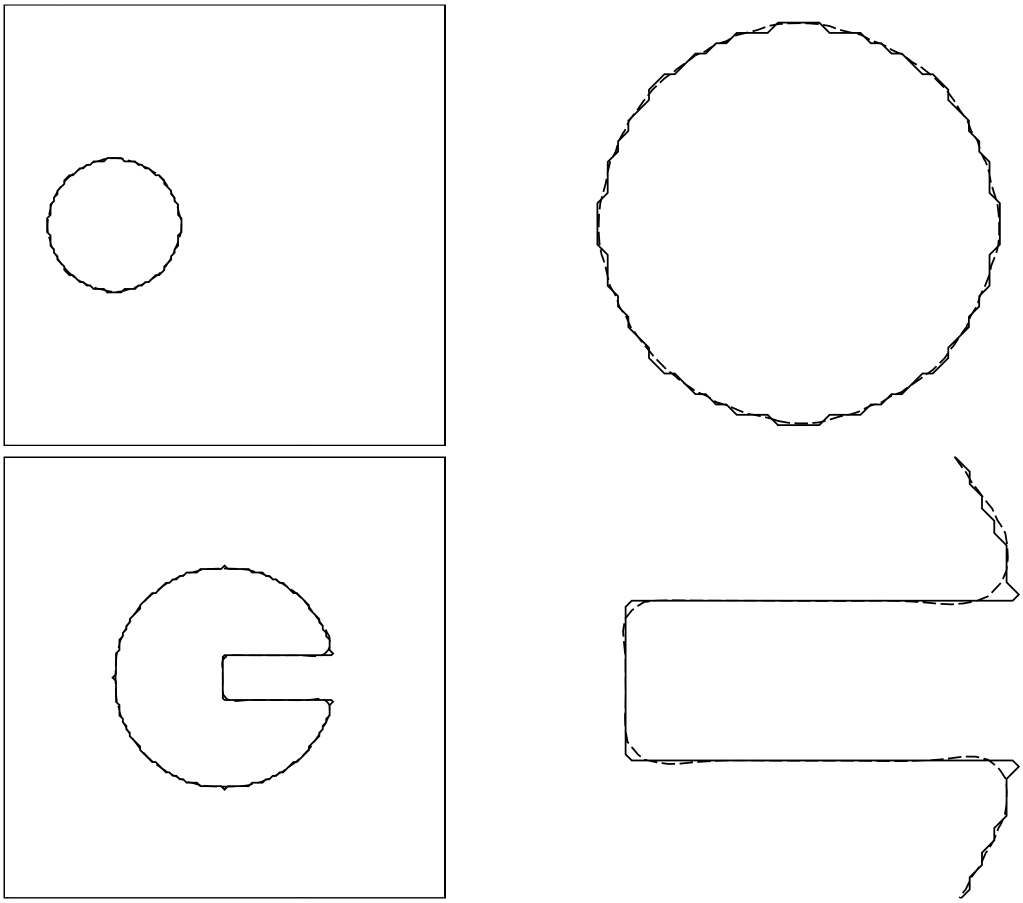

Figure 1: Topology of interface calculating the solution of tests 11b.a (top) and 11a.b (bottom) for n=0 (left) and (right). Initial and numerical solutions are plotted in solid and dashed lines respectively.

The estimate of the accuracy of the numerical advection methods is carried out according to the following criteria:

- the capacity to advect the field C without numerical diffusion, i.e. keeping a well defined interface between the two fluids,
- the conservation of mass M^*. The quantity

$$M^* = \sum_{i=2}^{N_x-1} \sum_{j=2}^{N_y-1} C_{i,j} \tag{3}$$

  must be perfectly conserved during numerical time. The indices i and j are related to the discretization points in x and y directions whereas N_x and N_y are the number of grid points in these directions. The relative variation can be calculated through

$$\Delta M^* = \sum_{i=2}^{N_x-1} \sum_{j=2}^{N_y-1} \frac{(C_{i,j} - C^0_{i,j})}{C^0_{i,j}} \tag{4}$$

  where $C^0_{i,j}$ is the initial concentration field.
- the accuracy of interface advection. The relative error E_C between the initial and final interface position, in the case when the concentration field is advected until it

recovers its initial position, can be calculated through the L_1 norm of the difference between the two solutions:

$$E_C = \frac{1}{(N_x - 2)(N_y - 2)} \sum_{i=2}^{N_x-1} \sum_{j=2}^{N_y-1} (C_{i,j}^n - C_{i,j}^0) \tag{5}$$

The superscript n refers to the last iteration of numerical solving.

4 Example of comparison exercise

On a 128 x 128 grid, the test cases 11a.a and 11a.b are solved by using an Eulerian Volume of Fluid approach, called VOF PLIC, based on a Piecewise Linear Interface Construction in each grid cell Youngs (1982). Figure 1 illustrates the initial and final solutions so obtained after one turn. A constant time step δt is chosen equal to 0.01 s corresponding to a final number of iterations, $n = 628$. After one turn, the mass conservation M^* equals to 10^{-16} for the two test cases whereas the relative error is $E_c = 0.001$ for test 11a.a and $E_c = 0.5$ for test 11a.b.

References

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