Test-case number 11b: Stretching of a circle in a vortex velocity field (N)

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1 Practical significance and interest of the test-case

The accuracy and overall quality of front tracking and front capturing methods is of major importance for fundamental and industrial research simulations devoted to multiphase flows. Two advection numerical tests dedicated to strong interface stretching are proposed here. Our objective is to estimate the sensitivity of interface tracking methods to free surface deformations and to quantify mass conservation in strongly varying interface shape problems. The first test-case considers a single vortex centered in a square box whereas in the second problem, a periodic multi-vortex velocity field is generated in a square cavity. In an analytical velocity field, an initially circular concentration shape of radius R_0 is distorted during n time iterations until interface structures of characteristic length less than $R_0/20$ are generated. Then, the flow field is reversed and a same calculation is performed during an equal duration to recover the initial cylindrical shape. After $2n$ iterations, the theoretical solution of the scalar advection problem is the initial interface condition. The works of Rider & Kothe (1995) or Rudman (1997) scrutinize numerical solutions provided by Volume Of Fluid, Level-Set or Front Tracking approaches on these two tests.

2 Definitions and physical model description

In two-phase flow modeling, the volume fraction or concentration C is a local function defined between 0 and 1 which is characteristic of the presence of one of the two fluids. The interface between the two phases is defined as $C = 0.5$. In an analytical velocity field $\mathbf{u}$, the advection of the volume fraction is solved through the passive scalar equation,

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = 0, \tag{1}$$

where t is the time variable.

3 Test-case description

The same initial conditions are used for the two test-cases. A cylindrical concentration of radius R_0 is initially centered at point (x_c, y_c) in a square cavity of length L with the following characteristics

- $R_0 = 0.15$ m,
- $L = 1$ m,
- $x_c = L/2$, $y_c = 3L/4$ for test 11b.a
- $x_c = L/2$, $y_c = L/2$ for test 11b.b

The velocity fields of the problems are respectively:

- Case 11b.a:

$$\mathbf{u}_a(x,y) = \begin{pmatrix} cos[\pi(x-L/2)] \; sin[\pi(y-L/2)] \\ -sin[\pi(x-L/2)] \; cos[\pi(y-L/2)] \end{pmatrix} \quad (2)$$

- Case 11b.b:

$$\mathbf{u}_b(x,y) = \begin{pmatrix} cos[4\pi(x+L/2)] \; cos[4\pi(y+L/2)] \\ sin[4\pi(x+L/2)] \; sin[4\pi(y+L/2)] \end{pmatrix} \quad (3)$$

4 Example of comparison exercise

The front tracking method of Shin & Juric (2002) is implemented to illustrate an almost analytical behavior of the test-cases thanks to markers put on the interface. With this approach, equation (1) is solved by advecting the markers in a lagrangian way. If $\mathbf{x}_i$ is the position of marker i, $\mathbf{u}$ the interface velocity and $\mathbf{n}_i$ the local unit normal to interface, the method reads as follows:

$$\frac{d\mathbf{x}_i}{dt} \cdot \mathbf{n}_i = \mathbf{u} \cdot \mathbf{n}_i \quad (4)$$

At each time step, the local volume fraction is then obtained by solving a Poisson equation:

$$\nabla^2 C = \nabla \cdot \int_{\Gamma(t)} \mathbf{n}_i \delta(\mathbf{x} - \mathbf{x}_i) d\gamma, \quad (5)$$

where $\delta(\mathbf{x} - \mathbf{x}_i)$ is a delta function that is non zero where $\mathbf{x} = \mathbf{x}_i$ and $\Gamma(t)$ is a time varying interface parameterized by the markers.

To illustrate the ability of the numerical method, a 100 x 100 grid is chosen to run the simulation with a time step $\Delta t = 0.005\ s$. 2000 iterations are calculated for both problems using equations (2) and (3) and 2000 further time steps with a reverse velocity field choosing $\mathbf{u} = -\mathbf{u}_a$ or $\mathbf{u} = -\mathbf{u}_b(x,y)$. In this way, after 4000 iterations, the initial cylindrical shape should be recovered, as presented in figure 1. Several authors have demonstrated that these test-cases are very difficult to achieve, in particular for Eulerian methods such as VOF-PLIC, VOF-TVD or Level-Set techniques (Rider & Kothe, 1995, Rudman, 1997, Vincent & Caltagirone, 2000).

References

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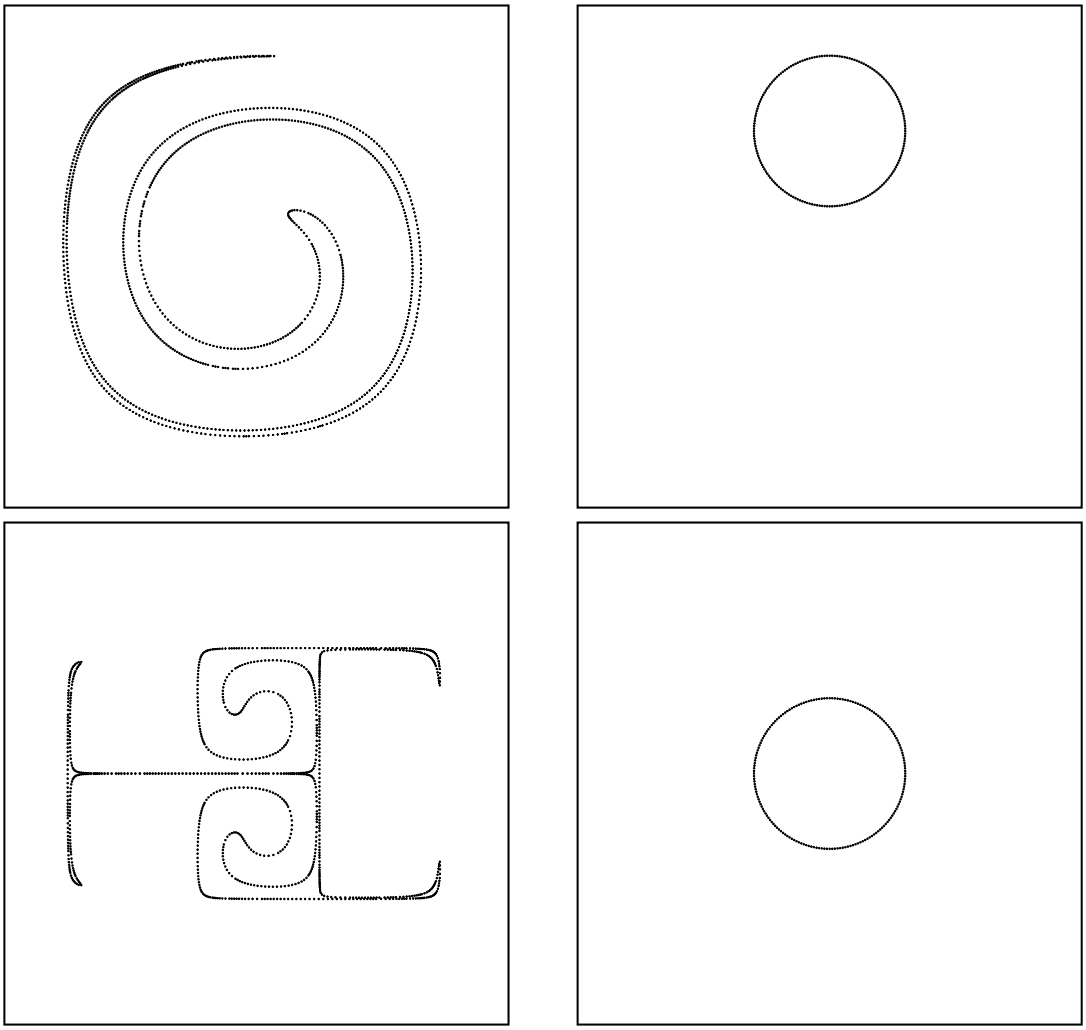

Figure 1: Front tracking simulation of test 11b.a (top) and 11b.b (bottom) after 2000 (left) and 4000 (right) iterations. The marker positions are plotted.

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