Test-case number 13: Shock tubes (PA)

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1 Introduction

We are interested here in test-cases about two-fluid compressible flows. An analytical solution is given by the exact solution of the Riemann Problem associated with the system of balance equations describing the behavior and the motion of the flow (Rouy, 2000, Barberon *et al.* , 2003b,a).
The hypotheses considered are the following:

- both fluids of the flow are separated by a topologically simple interface,
- the flow is compressible, inviscid, with no chemical reaction and no heat exchange,
- the surface tension and the gravity effects are neglected.

First, we propose one-dimensional analytical results for a gas-gas flow, then one-dimensional analytical results for a gas-liquid flow are also shown.

2 The mathematical model and the solution of the corresponding Riemann Problem

The mixture of both fluids is supposed to be a continuous medium: density, momentum and energy can be defined at each point of the domain. These functions are not necessarily continuous. According to the physical hypotheses described below, the equations to take into consideration are the balance equations for a compressible flow: balance of mass ρ (1), momentum $\rho\overrightarrow{V}$ (2) and specific total energy ρE (3)

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \overrightarrow{V}) = 0, \tag{1}$$

$$\frac{\partial (\rho \overrightarrow{V})}{\partial t} + \overrightarrow{\mathrm{div}}(\rho \overrightarrow{V} \otimes \overrightarrow{V}) + \overrightarrow{\mathrm{grad}} P = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho E) + \mathrm{div}((\rho E + P)\overrightarrow{V}) = 0. \tag{3}$$

In order to solve this system, a closure relation must be used. When dealing with a single fluid, it is classical to use an equation of state $P = P(\rho, e)$ which links pressure to density and to internal energy e with $E = e + 1/2||\overrightarrow{V}||^2$. The perfect gas equation $P = (\gamma - 1)\rho e$ where $\gamma = c_p/c_v$ denotes the ratio of the specific heat capacities, is one of these. Other relations exist. For example, the Stiffened-gas equation

$P = (\gamma - 1)\rho e - \gamma P_\infty$ can be used for different types of materials. The constants γ and P_∞ depend on the considered material and are determined by experimental measurements. This relation has been chosen because it is relevant for liquids and gases. The coefficients γ and P_∞ must be in such a way that they respect the nature of the fluid. The pressure of the medium does not only depend here on the density and the internal energy but also depends on the location of the interface Γ between the two fluids. In fact the interface is seen as a discontinuity of the physical properties of the continuous medium.

Consequently to determine the pressure and to close the system, we need to track the interface.

The interface moves at the velocity of the continuum. If we define a quantity denoted ϕ which determines its location, this quantity verifies the following equation,

$$\frac{\partial \phi}{\partial t} + \overrightarrow{V}.\overrightarrow{\mathrm{grad}}\phi = 0. \tag{4}$$

Thanks to the mass balance (1), this convection equation can be rewritten in a conservative form,

$$\frac{\partial (\rho\phi)}{\partial t} + \mathrm{div}(\rho \overrightarrow{V} \phi) = 0. \tag{5}$$

Several choices of ϕ are possible (volume fraction, level set function, etc.). They correspond to different types of numerical methods for tracking interfaces: VOF methods (Hirt & Nichols, 1979), front tracking methods (Sethian *et al.* , 1992, Glimm, 1986) or multifluid methods (Saurel & Abgrall, 1999).

Finally, we obtain the following system,

$$\frac{\partial U}{\partial t} + \frac{\partial f(U)}{\partial x} = 0, \quad x \in \mathbb{R}, t > 0. \tag{6}$$

with $U = (\rho, \rho u, \rho E, \rho\phi)^T$, $x \in \mathbb{R}, t > 0$, $f(U) = (\rho u, \rho u^2 + P, (\rho E + P)u, \rho\phi u)^T$, $E = e + \frac{1}{2}u^2$ and the closure relation $P = (\gamma(\phi) - 1)\rho e - \gamma(\phi)P_\infty(\phi)$

Definition 1 *The one-dimensional Riemann Problem associated to the system (6) is the following Cauchy Problem,*

$$\frac{\partial U}{\partial t} + \frac{\partial f(U)}{\partial x} = 0, \quad x \in \mathbb{R}, t > 0 \tag{7}$$

$$U(x,0) = \left\{ \begin{array}{ll} U_L, & x < 0 \\ U_R, & x > 0 \end{array} \right.$$

It is well known that this system admits a weak auto-similar solution (function of x/t), physically admissible which consists in a succession of constant states separated by entropic shock waves, rarefaction waves or contact discontinuities. This solution is unique and is shown in figure 1.

According to Rouy (2000), solving the Riemann Problem (7) is equivalent to solve the above equation,

$$u_L - u_R - \Xi^L(P) - \Xi^R(P) = 0, \tag{8}$$

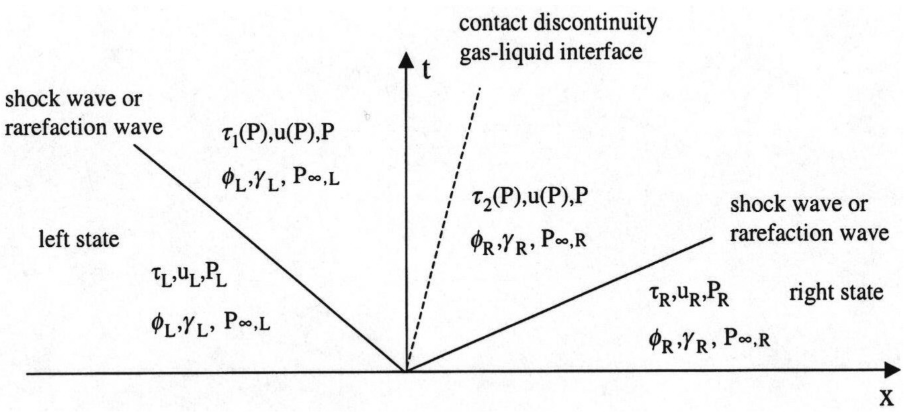


Figure 1: The solution of the Riemann problem

where

$$\Xi^i(P) = \begin{cases} \Phi^i(P), & \text{if } P > P_i \text{ (shock wave)} \\ \Psi^i(P), & \text{if } P < P_i \text{ (rarefaction wave)} \end{cases} \tag{9}$$

with,

$$\Phi^i(P) = \frac{(P - P_i)\sqrt{2\tau_i}}{\sqrt{(\gamma_i + 1)P + (\gamma_i - 1)P_i + 2\gamma_i P_{\infty,i}}} \tag{10}$$

and,

$$\Psi^i(P) = 2\frac{\sqrt{\gamma_i \tau_i}}{\gamma_i - 1}\sqrt{P_i + P_{\infty,i}}\,((\frac{P + P_{\infty,i}}{P_i + P_{\infty,i}})^{\frac{\gamma_i - 1}{2\gamma_i}} - 1) \tag{11}$$

τ denoting the covolume *i.e.* the inverse of the density, $i = L$ for the left-hand side wave and $i = R$ for the right-hand side wave.

The solution of the equation (8) for P is obtained by Newton's method. Then, we determine τ_1 and τ_2 with the following relations:

- if $P > P_L$, we have a 1-shock, $\tau_1 = h_s^L(P)$
- if $P < P_L$, we have a 1-rarefaction wave, $\tau_1 = h_r^L(P)$
- if $P > P_R$, we have a 3-shock, $\tau_2 = h_s^R(P)$
- if $P < P_R$, we have a 3-rarefaction wave, $\tau_2 = h_r^R(P)$,

with

$$h_r^i(P) = \tau_i(\frac{P_i + P_{\infty,i}}{P + P_{\infty,i}})^{\frac{1}{\gamma_i}} \tag{12}$$

and

$$h_s^i(P) = \tau_i \frac{(\gamma_i + 1)P_i + (\gamma - 1)P + 2\gamma_i P_{\infty,i}}{(\gamma_i + 1)P + (\gamma_i - 1)P_i + 2\gamma_i P_{\infty,i}} \tag{13}$$

Finally, the velocity of the contact discontinuity, u, can be obtained for example by the following relation,

$$u(P) = u_L - \Xi^L(P) \tag{14}$$

The indexes L or R show us from which constant state (Left or Right) the functions Ξ and h are depending.

Remark 1 *In Rouy (2000), Barberon* et al. *(2003a), we have demonstrated that the Riemann Problem admits a global solution* i.e. *even in the case of vacuum apparition. Physically, this corresponds to a null pressure (in the particular case of vacuum apparition in gas) or to a negative pressure (in the particular case of cavitation in water).*

3 The shock tube

This is a one-dimensional test-case. We consider a tube with an arbitrary length (here L=8 m) initially filled with two fluids separated by a membrane (see figure 2). We suppose that the membrane breaks and we observe the evolution. The interface between both fluids moves. The thermodynamical and kinematical quantities change. This solution is given by the solution of the Riemann Problem presented above.

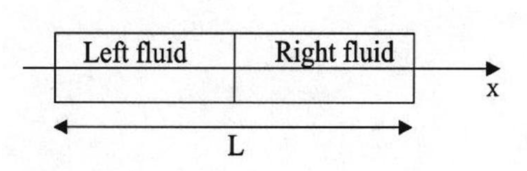


Figure 2: The shock tube

We propose 4 test-cases :

1. Both parts of the tube are initially filled with the same gas, with different physical characteristics detailed in table 1.

	Left state	Right State
ρ	1 kg/m^3	2 kg/m^3
u	0 m s^{-1}	0 m s^{-1}
P	1.2 bar	1 bar
γ	1.4	1.4
P_∞	0 bar	0 bar

Table 1: Parameters for test-case 1

2. The left part contains helium and the right part contains air. Both gases have null velocity and have the same pressure (test-case of stationary contact discontinuity, see table 2).

3. The previous test is modified in such a way that the ratio of the two pressures increases, the velocity of helium is now equal to 10 m/s (see table 3).

	Left state	Right State
ρ	0.192 kg/m^3	1.156 kg/m^3
u	0 m s^{-1}	0 m s^{-1}
P	1 bar	1 bar
γ	1.667	1.4
P_∞	0 bar	0 bar

Table 2: *Test-case 2*

	Left state	Right State
ρ	0.192 kg/m^3	1.156 kg/m^3
u	10 m s^{-1}	0 m s^{-1}
P	10 bar	1 bar
γ	1.667	1.4
P_∞	0 bar	0 bar

Table 3: Test-case 3

4. The last test deals with a gas-liquid flow with a ratio of pressure equal to 100, the initial conditions are shown in table 4.

	Left state	Right State
ρ	10 kg/m^3	1000 kg/m^3
u	10 m s^{-1}	0 m s^{-1}
P	100 bars	1 bar
γ	1.4	5.5
P_∞	0 bar	4900 bars

Table 4: Test-case 4

The calculated results to be considered as the reference solution to these test-cases are shown in figures 3,4, 5 and 6.

References

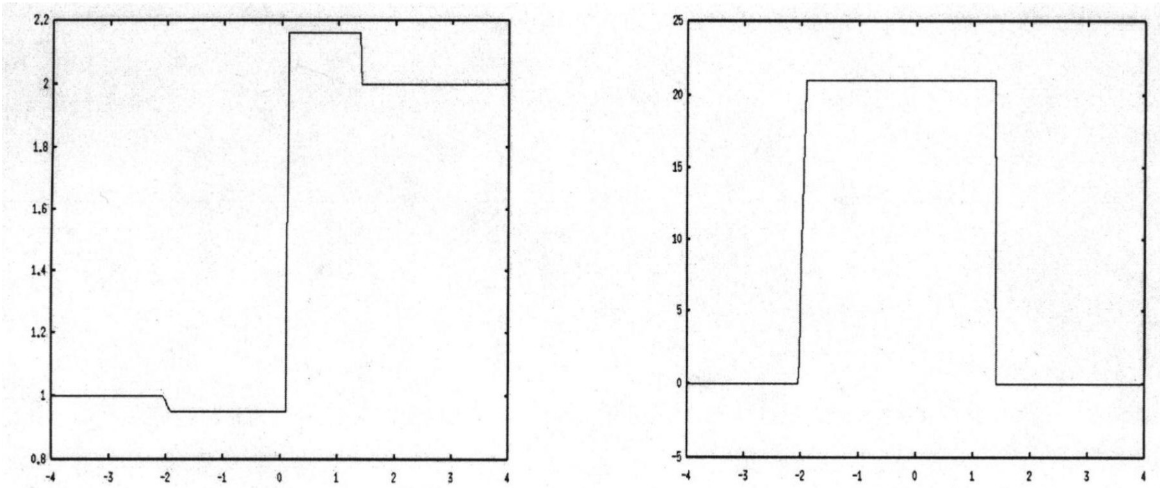
Barberon, T., Helluy, P., & Rouy, S. 2003a. Apparition du vide - Méthode à deux flux. *Computers and Fluids to appear.*

Barberon, T., Helluy, P., & Rouy, S. 2003b. Practical computation of axisymmetrical multifluid flows. *International Journal of Finite Volumes.*

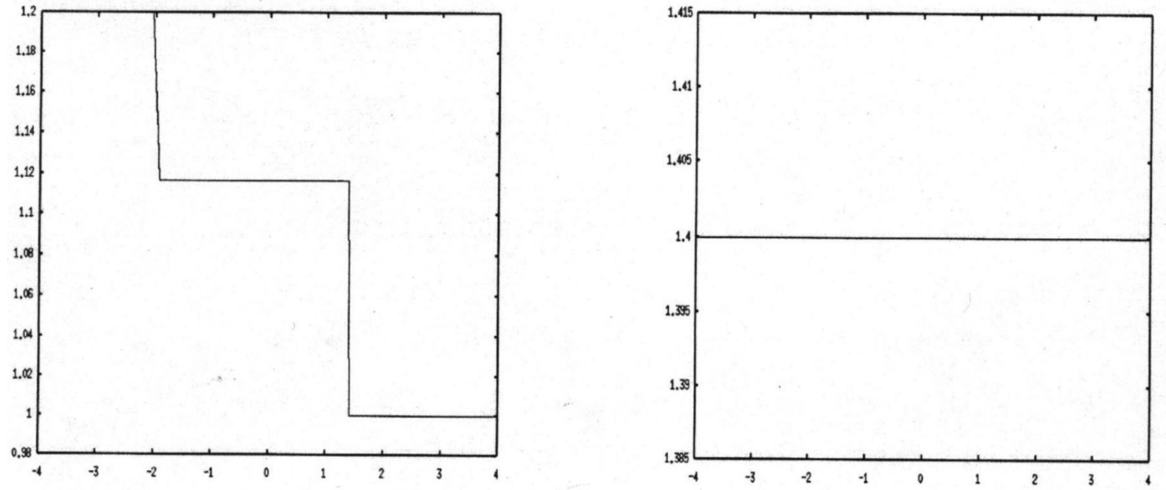
Glimm, J. 1986. Front tracking applied to Rayleigh-Taylor instability. *Physics of fluids*, **10**.

Hirt, C., & Nichols, B. 1979. VOF Method for the dynamics of free boundaries. *Journal of Computational Physics*, **39**, 201–225.

Rouy, S. 2000. *Modélisation mathématique et numérique d'écoulements diphasiques compressibles - Application au cas industriel d'un générateur de gaz.* Ph.D. thesis, Université de Toulon et du Var, France.

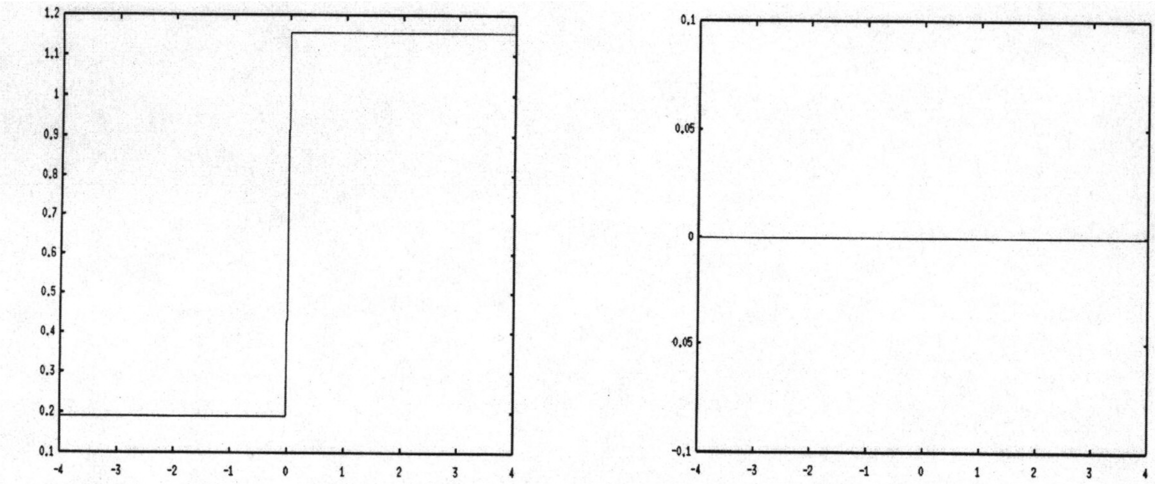


(a) *Density (kg/m^3) and Velocity (m s^{-1})*

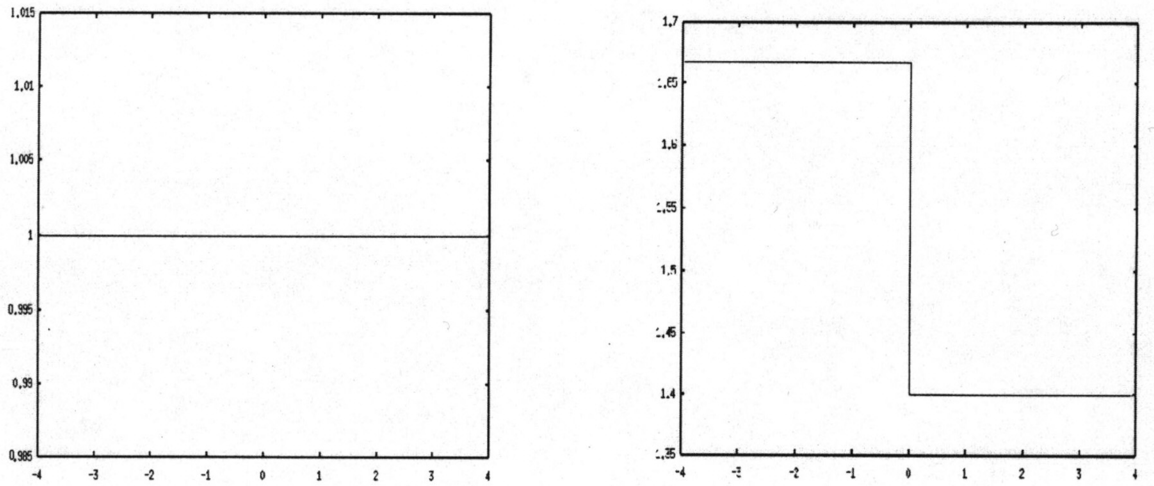


(b) *Pressure (bar) and* γ

Figure 3: Shock tube 1, gas-gas, time=5 ms, CFL=0.18, 400 cells.

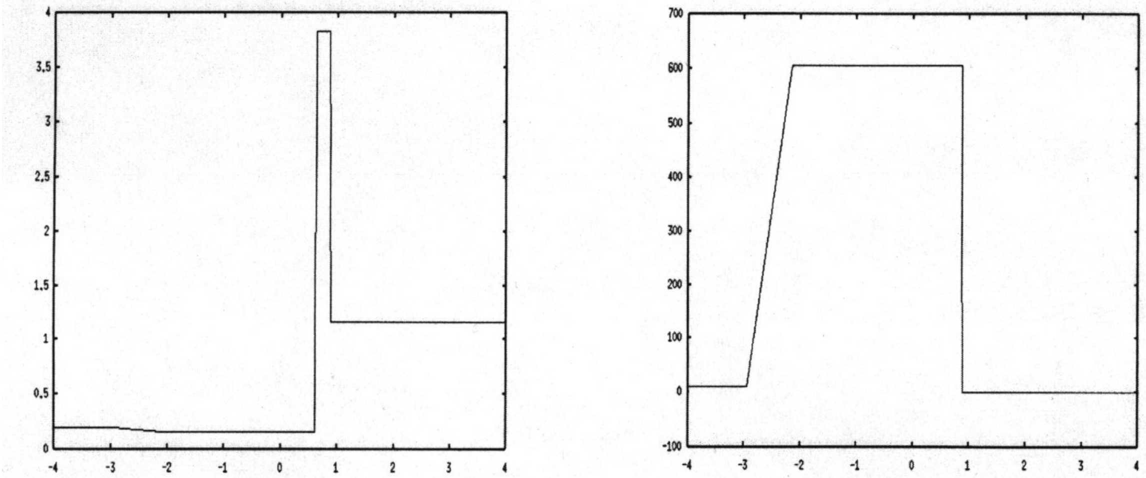


(a) *Density (kg/m^3) and Velocity (m s^{-1})*

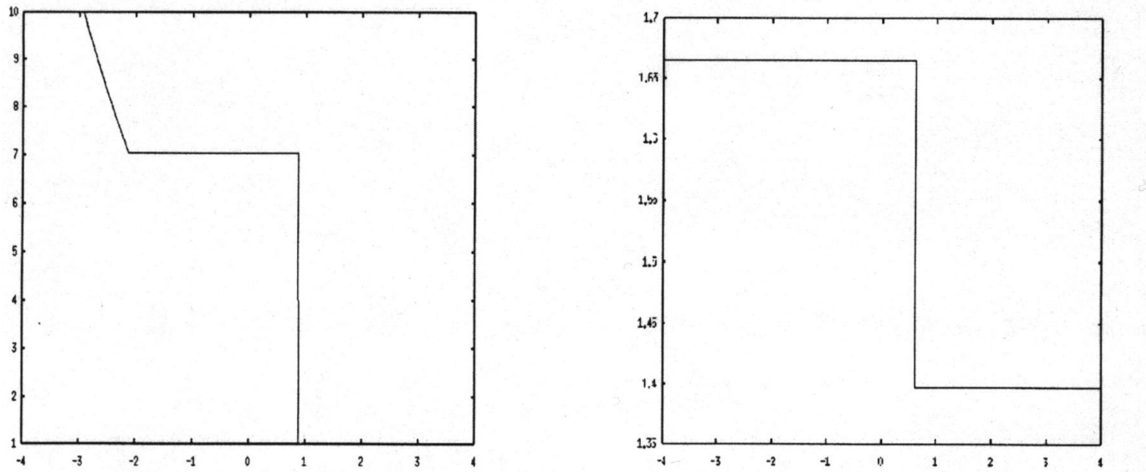


(b) *Pressure (bar) and γ*

Figure 4: Shock tube 2, helium-air, time=5 ms, CFL=0.39, 400 cells.

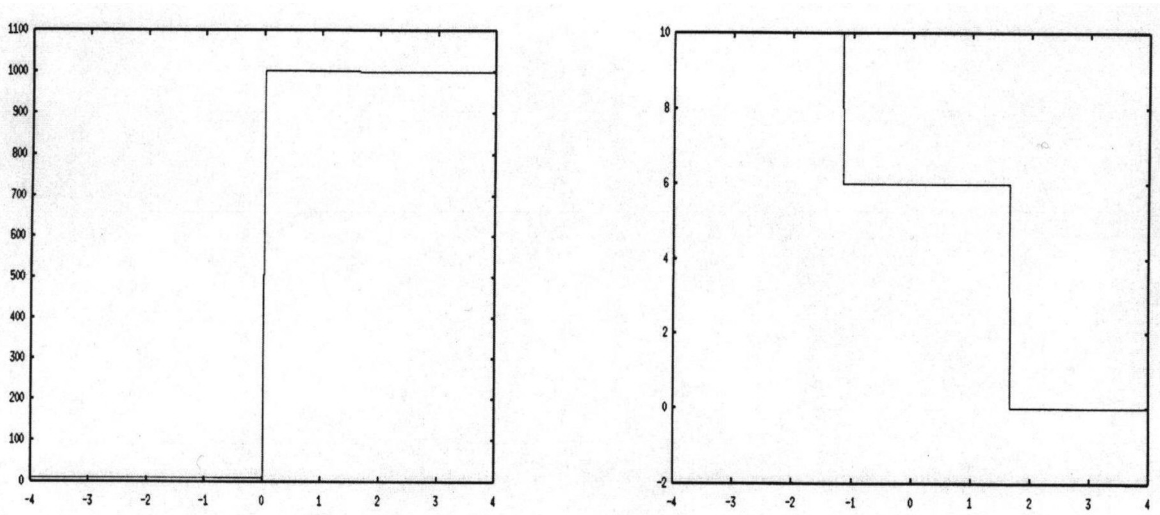


(a) *Density (kg/m^3) and Velocity (m s^{-1})*

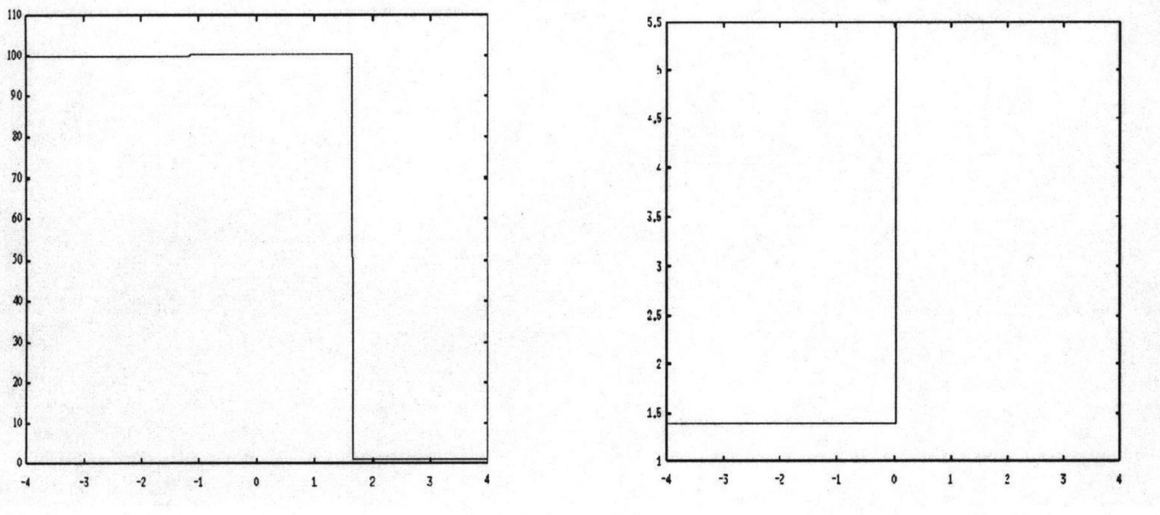


(b) *Pressure (bar) and* γ

Figure 5: Shock tube 3, helium-air, time=1 ms, CFL=0.28, 400 cells.



(a) *Density (kg/m^3) and Velocity (m s^{-1})*



(b) *Pressure (bar) and γ*

Figure 6: Shock tube 4, gas-liquid, time=1 ms, CFL=0.14, 400 cells.

Saurel, R., & Abgrall, R. 1999. A simple method for compressible multifluid flows. *SIAM Journal of Scientific Computing*, **21**, 1115–1145.

Sethian, J., Mulder, W., & Osher, S. 1992. Computing interface motion in compressible gas dynamics. *Journal of Computational Physics*, **100**, 209–228.