Test-case number 14: Poiseuille two-phase flow (PA)

March 2003

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1 Practical significance and interest of the test-case

The two-phase Poiseuille flow is a simple interfacial flow that permits to estimate accurately the time and space convergence order of the numerical resolution of the Navier-Stokes equations in their Eulerian two-phase flow formulation. Moreover, this test case allows characterizing the sensitivity of the numerical solution with respect to the averages implemented on the density and the viscosity at the interface in the discretization of the motion equations. To finish with, the Poiseuille flow allows calculating analytically the viscous stress tensor to verify if the continuity of its tangential component is verified numerically at the interface.

The present test case is interesting because it possesses a theoretical solution. However, no interface deformation is induced in this problem. Therefore, it represents a necessary test, but certainly not a sufficient reference.

2 Definitions and physical model description

The horizontal stratified flow of a two fluid between two parallel walls is considered (see figure 1). The gravity and the surface tension forces are neglected. For long times, a steady solution is obtained for the two-phase Poiseuille flow problem which can be described by an analytical solution. If L is the length of the horizontal walls, d is the distance between the bottom horizontal boundary and the interface and H the distance between the two horizontal boundaries, the velocity field, $\mathbf{u} = (u_x, u_y)$, and the pressure, p, can be calculated by assuming the velocity to be parallel to the x axis and by considering the continuity of the velocity and the viscous stress tensor at the interface. In this way, the velocity field, $\mathbf{u}_1(u_{x,1}, u_{y,1})$ and $\mathbf{u}_2(u_{x,2}, u_{y,2})$, respectively in each fluid are given by the

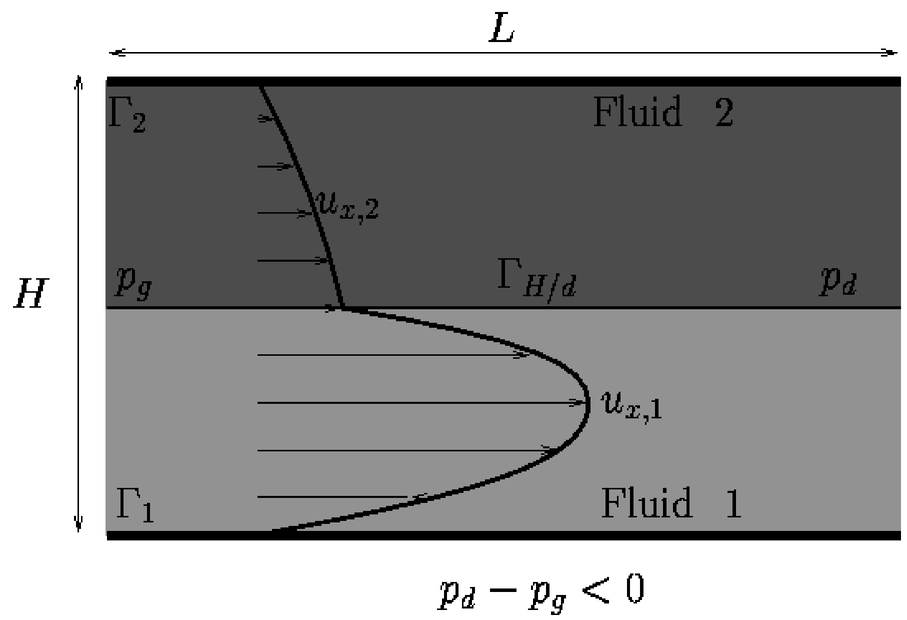

Figure 1: Two-phase Poiseuille flow between to parallel walls

following equations (Coutris *et al.* , 1989, Vincent, 1999)

$$u_{x,1}(x,y) = \frac{\Delta p(d[\mu_2 - \mu_1(1-d)]y^2 - H[\mu_2 - \mu_1(1-d^2)]y)}{2\mu_1 d(\mu_2 - \mu_1[1-d])} \quad (1)$$

$$u_{y,1}(x,y) = 0 \quad (2)$$

$$u_{x,2}(x,y) = \frac{\Delta p(d[\mu_2 - \mu_1(1-d)]y^2 - H[\mu_2 - \mu_1(1-d^2)]y - H^2[\mu_2 d + \mu_1(1-d)])}{2\mu_2 d(\mu_2 - \mu_1[1-d])} \quad (3)$$

$$u_{y,2}(x,y) = 0 \quad (4)$$

$$p(x,y) = \frac{(p_r - p_l)}{L}x + p_l \quad (5)$$

where $\Delta p = p_r - p_l$ is the pressure difference between the outlet and the inlet boundary pressure.

3 Test-case description

The flow characteristics are defined by

- $\Delta p = -0.212435\ Pa$,
- $H/d = 0.5$,
- $1 \leq \rho_2/\rho_1 \leq 1000$,
- $1 \leq \mu_2/\mu_1 \leq 5000$,
- $\Delta H/512 \leq x \leq H/2$.

No-slip boundary conditions are imposed on the horizontal walls whereas Neumann conditions are assumed on the velocity on the inlet and outlet boundaries. The pressures p_l and p_r are imposed on the left and right limits of the calculation domain to ensure the pressure difference Δp. As an example, the comparison between the analytical

N	Absolute error
2	$0.2262 \cdot 10^-15$
32	$0.6921 \cdot 10^-15$
512	$0.7679 \cdot 10^-13$

Table 1: Evolution of the absolute error on velocity with respect to the grid resolution.

and the numerical velocity fields implemented with a Volume Of Fluid (VOF) numerical model (Vincent & Caltagirone, 2000) is presented in figure 2 with $H = 0.02m$, $\mu_1 = 5 \cdot 10^{-4} Pa.s$ and $\mu_2 = 1.85 \cdot 10^{-5} Pa.s$. Whatever the grid resolution N in each direction (from 2 to 512 discretization points in the cross flow direction), the L_2 absolute error is almost equal to the truncation error (see table 1).

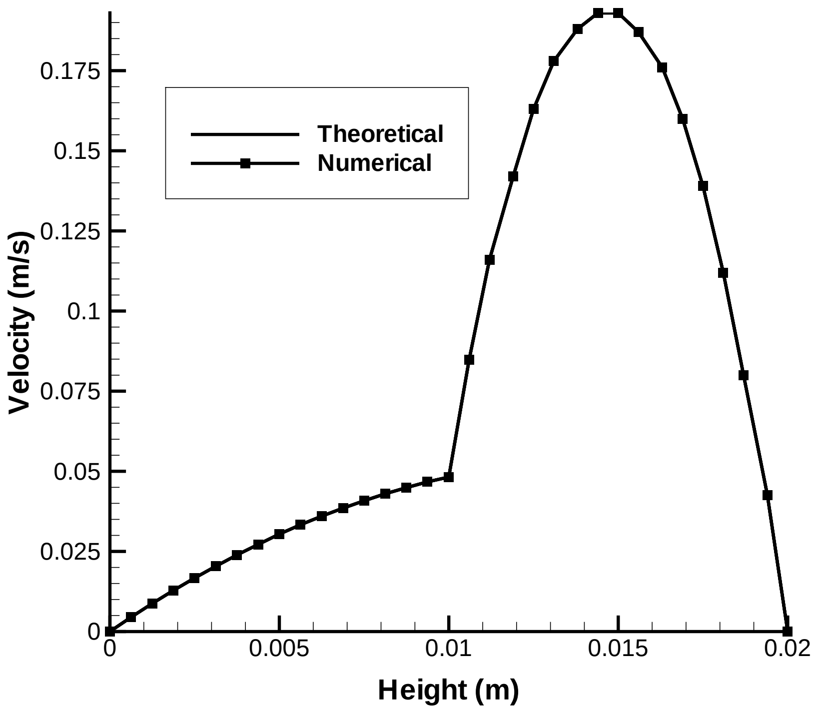

Figure 2: VOF simulation of the two-phase Poiseuille flow between to parallel walls on a 32 x 32 grid for $H = 0.02m$, $\mu_1 = 5 \cdot 10^{-4} Pa.s$ and $\mu_2 = 1.85 \cdot 10^{-5} Pa.s$ - Comparison between numerical and theoretical solutions.

References

Coutris, N., Delhaye, J.M., & Nakach, R. 1989. Two-phase flow modelling: the closure issue for a two-layer flow. *Int. J. Multiphase Flow*, **15**, 977–983.

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