Test-case number 19: Shock-Bubble Interaction (PN)

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1 Introduction

We describe here a test proposed by Quirk & Karni (1996) based on the experiments of Haas & Sturtevant (1987). Its goal is to simulate the propagation of a shock through a helium bubble in air. Haas and Sturtevant initial purpose was to get a better understanding of the Richtmeyer-Meshkov instabilities. More generally they wanted to understand how pressure waves in heterogeneous media can generate turbulent phenomena which tend to mix the fluids. From a numerical point of view the goal of this test is to validate compressible multifluid flows models as well as numerical methods used for solving these models. This test has been performed at least in the following studies by Abgrall (1996), Fedkiw *et al.* (1999), Karni (1996), Kokh & Allaire (2001) and Saurel & Abgrall (1999).

2 Description

Geometry. The test is two-dimensional. The computational domain is a rectangular box which is 890 mm long (horizontal axis) and 89 mm high (vertical axis). At time $t = 0$, the bubble has a 50 mm diameter and its center is located at (x_c, y_c), $x_c = 420$ mm, $y_c = 44.5$ mm (the origin being the low left corner of the domain). The initial location of the shock is a vertical line which is 222.5 mm away from the right side of the domain.

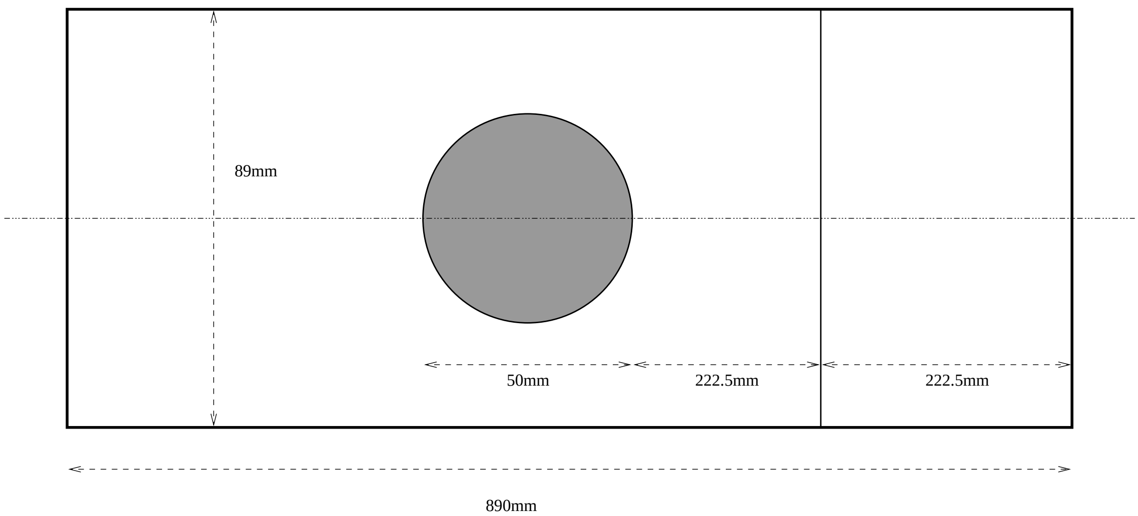

Figure 1: Computational domain.

Physical Model. The behavior of each fluid is governed by the gas dynamics compressible Euler equations (without any diffusion term, neither surface tension, nor gravity).

Gas	γ	R (kJ.kg^{-1}.K^{-1})	C_v (kJ.kg^{-1}.K^{-1})
Air	1.4	0.287	0.72
Helium	1.648	1.578	2.44

Table 1: Equation of state parameters.

zone	density	pressure	internal energy	velocity
units	(kg.m^{-3})	(bar)	(10^5J.kg^{-1})	(10^3m.s^{-1})
post-shock air (right side)	1.376363	1.569800	2.851355	(-0.394728 ; 0.0)
pre-shock air (left side)	1.0	1.0	1.0	(0.0 ; 0.0)
helium bubble	0.181875	1.0	8.48500	(0.0 ; 0.0)

Table 2: Initial state.

Each fluid is assumed to obey the perfect gas equation of state. Thus the fluid $i = 1, 2$ is modeled by the following equations,

$$\begin{cases} \partial_t \rho_i + \text{div}\,(\rho_i u_i) = 0 \\ \partial_t\,(\rho_i u_i) + \text{div}\,(\rho_i u_i \otimes u_i + p_i \text{I}) = 0 \\ \partial_t\,(\rho_i e_i) + \text{div}\,[(\rho_i e_i + p_i) u_i] = 0 \end{cases} \tag{1}$$

where ρ_i is the density, u_i the velocity (it is a two-component vector), e_i the specific total energy such that $e_i = \varepsilon_i + |u_i|^2/2$ with ε_i the specific internal energy, the pressure p_i being provided by,

$$p_i = (\gamma_i - 1)\rho_i \varepsilon_i, \tag{2}$$

where γ_i is the ratio of the heat capacities of the ith gas. The interface modeling and its numerical treatment are free choices (most of the simulation references use an isothermal-isobaric mixture law in order to thicken the interface which is then captured on an Eulerian mesh).

Numerical Data. The equation of state parameters for air and the helium bubble are provided in table 1 (data for R and C_v are, *a priori*, not required). Let us note that the parameters for the helium bubble describe indeed a 28% mass mixture between helium and air. The shock travels from the right side of the domain to the left side with a 1.22 Mach velocity. This means its velocity is 1.22 times higher than the sound velocity in the pre-shock air at rest (atmospheric pressure and density equal to 1 kg.m^{-3}). Let us recall that for a perfect gas, the sound velocity is given by $c = \sqrt{\gamma p/\rho}$ and that the horizontal shock velocity is -1.443523×10^3 m.s^{-1}. The helium bubble is supposed to be initially at mechanical equilibrium with the surrounding air. Thanks to the Rankine-Hugoniot conditions it is possible to find the initial values for air after the shock. Using compatible units with those used in the equation of state, the initial state is defined in table 2 (let us recall that: 1 bar $= 10^5$ Pa, and Pa $= 1$ J m^{-3}).

Boundary conditions. The horizontal boundaries of the domain are solid walls where "mirror" boundary conditions are to be applied (*i.e.* non-penetration conditions). The right vertical boundary is set to be an "inflow" boundary condition equal to the initial data for air to the right of the shock. The left vertical boundary is treated as a free "outflow", which means a zero order extrapolation of the variables has to be performed out of the computational domain.

Measures and Comparisons. It is required to compare the shape of the bubble with Haas and Sturtevant experimental results at the following time steps: 32, 52, 62, 72, 82, 102, 245, 427, and 674 μs. We shall also plot the pressure evolution in time downstream from the position of the bubble.

Remark. Data provided in Fedkiw *et al.* (1999) are different from those in Quirk & Karni (1996) which are being used here. There exists a similar test where the helium is replaced with a gas heavier than air (refrigerant fluid R22, see Quirk & Karni, 1996))

Remark. Let us note that similar tests are available (see *e.g.* Allaire *et al.* , 2002, Shyue, 1999), however no direct comparison is possible as these tests deal with different equations of state than those which are proposed here.

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