Test-case number 21: Gas bubble bursting at a free surface, with jet formation (PN-PÉ)

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1 Practical significance and interest of the test-case

The bursting of a gas bubble, initially in equilibrium under a gas/liquid surface, is numerically studied. Computation takes place in a 3D axisymmetric geometry. Gravity and surface tension forces lead to a focus of surface waves and, depending on the problem parameters, produce an upward liquid jet. Usually, numerical simulation is stopped when the pinching of this jet arises, just before creating the first liquid droplet; however, in the present case, the code is able to follow few ejected droplets.

The present test-case can be used to verify the precision of the interface position as well as the treatment of surface tension forces, which are preponderant here. Extensive simulations show a great sensitivity to initial conditions. Initial shape, as described in table I, must be scrupulously respected; if not, different results (especially in the shape and speed of the upward liquid jet) would be obtained.

2 Definitions and physical model description

A potential (irrotational) viscous flow model is used. Inertia, gravity and surface tension forces are computed without any approximation, while viscous forces are reduced to normal stress on the interface (no shear stress at the interface). More details can be found in Georgescu et al. (2002).

Before the bursting (i.e. when \( t < 0 \)), the gas bubble is in equilibrium and its shape is computed from a theoretical model (Ivanov et al., 1986), where the upper liquid film is supposed to be thin. In the case where the bubble radius is small (e.g. less than 1 mm for the air/water couple), this shape is well approximated by a sphere (see figure 2).

At time \( t = 0 \), the upper liquid film is removed, leading to an open cavity, whose shape is smoothed by a spline interpolation (cubic piecewise splines), as shown in figure 2. The \((r, z)\) coordinates of this initial shape are shown in table I.

3 Test-case description

Let’s denote \( R_0 \) the initial radius of the bubble, \( \sigma \) the liquid/gas surface tension, \( g \) the gravity, \( \rho_L \) the liquid density and \( \mu_L \) the viscosity. In order to obtain a nondimensional framework, the following scales are chosen :
Figure 1: Theoretical initial shape of the bubble: air/water. $R_0 = 0.75 \, mm$, $Fr = 13.2$, $Re = 233$

Figure 2: Practical initial shape of the bubble: initial condition for the computation.

- length: $R_0$
- velocity: $\sqrt{\frac{\sigma}{R_0 \rho_L}}$
- pressure: $\frac{\sigma}{R_0}$

The nondimensional parameters are then:

- Weber: $We = 1$ (always, due to the choice of reference scales)
- Froude: $Fr = \frac{\sigma}{\rho_L g R_0^2}$
### Table 1: Nondimensional coordinates of points on figure 2.

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- Reynolds: $Re = \sqrt{\frac{R_0 \sigma \rho_L}{\mu_L}}$

Computational results will then use the dimensionless length $l^* = \frac{l}{R_0}$ and the dimensionless time $t^* = t \sqrt{\frac{\sigma}{\rho_L R_0^3}}$. The size of the computational domain is defined by:

$$0 < r^* < 4, \quad z^* > -4$$

Comparison may be done on the upward jet speed (fig. 3), on the interface shape at selected time values (fig. 4), on the radius of the first ejected droplet, and on the critical bubble radius for which the liquid jet decays without ejecting a liquid droplet (cf. Georgescu et al. (2002)).

For experimental comparisons, see Kientzler et al. (1954), and Suzuki & Mitachi (1995). Photographic sequences taken from Kientzler et al. (1954) are depicted on figures 5 and 6 for an air bubble bursting in fresh water. (N.B. The three small bubbles visible on each frame of fig. 6 don’t depend on the bursting process; it seems that they are attached to the glass wall through which photos have been taken.)

Boulton-Stone & Blake (1993) also present numerical results with the same kind of method as used here.

**References**

Figure 3: Upward liquid jet velocity vs time. $R_0 = 0.75$ mm, $Fr = 13.2$, $Re = 233$.

Figure 4: Free-surface time evolution after bursting bubble. $R_0 = 0.75$ mm, $Fr = 13.2$, $Re = 233$. 
Figure 5: Bursting bubble (after Kientzler et al. [1954], fig. 1, p.3), $R_0 = 0.5$ mm. View angle is 10 degrees above horizontal plane. The time interval between frames is about 0.3 ms.

Figure 6: Bursting bubble (after Kientzler et al. [1954], fig. 2, p.4), $R_0 = 0.75$ mm. Photos taken through a glass wall. The time interval between frames is about 0.3 ms.


