Test-case number 24: Growth of a small bubble immersed in a superheated liquid and its collapse in a subcooled liquid (PE,PA)

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1 Practical significance and interest of the test-case

This test-case describes an analytical solution of a series of simple free boundary problems. Firstly, the growth of a vapor bubble initially at rest, in mechanical and local thermal equilibrium with the superheated liquid. Next, the collapse of a vapor bubble immersed in a subcooled liquid is considered. In this latter situation, the bubble is initially at rest and in mechanical equilibrium only with the liquid phase.

The theory that provides the reference solution is that of Plesset & Zwick (1954) who discussed thoroughly its validity domain. An experiment is also proposed to strengthen the confidence to be put in their model. It is claimed by these authors that experimental conditions are consistent with the theory. This is confirmed by their analysis of the problem scales. It is therefore proposed three test-cases selected from their work.

- Inertia controlled collapse of a bubble, dubbed the Rayleigh regime, where the heat flux from the liquid to the bubble is irrelevant to the problem.

- The initial stage of the growth of a vapor bubble where both surface tension and transient heat flux to the bubble interface governs the dynamics of the phenomenon.

- The long-term growth of the same bubble, which is only controlled by the rate of heat transfer from the liquid to the interface.

The analytical solutions to be described here have been obtained by coding an improved version (variable time step) of the original algorithm proposed by Zwick & Plesset (1954). The heat transfer model in the liquid phase describes both the convection and the conduction. It is solved in closed form with a slight approximation (Plesset & Zwick, 1952). The main assumption to get this solution is the spherical symmetry, which might be questionable for the final stage of a bubble collapse.

Finally, coupling the heat transfer problem to the motion of the interface results in a non-linear integral-differential problem for which Zwick & Plesset (1954) proposed a solution algorithm. In their original paper there are few misprints which make the material useless for practical calculations. Lemonnier (2001) revisited this work and proposed a corrected version of the theory validated by a comparison of numerical calculations with the results of the original paper. Moreover, the original work of Plesset & Zwick (1954) compared the original model with experiments. One of them is selected as an experimental test-case.

2 Model and assumptions

The assumptions of the model are the following.

H1. The liquid and its vapor are not compressible.

H2. The liquid and vapor viscosity are neglected.

H3. The vapor enclosed by the bubble is assumed to have uniform thermodynamic properties and is in thermodynamic equilibrium with the liquid at the interface. The only exception to this assumption is relative to the density of the vapor, which is allowed to vary with time but however remains in saturated state corresponding to the interface temperature.

H4. The physical and transport properties of the liquid are uniform and constant.

H5. Convection that would be expected from the bubble buoyancy is neglected (no gravity).

H6. At any time, the system remains spherically symmetric.

Under these circumstances, it can be shown that the time evolution of the bubble radius is given by,

$$R\frac{\mathrm{d}^2R}{\mathrm{d}t^2}+\frac{3}{2}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2=\frac{1}{\rho_L}\left(p_v(T)-p_{L\infty}-\frac{2\sigma}{R}\right), \tag{1}$$

where R is the bubble radius, t is time, ρ_L is the liquid density, T is the interface temperature, $p_v(T)$ is the saturation pressure of the liquid evaluated at the temperature of the interface, $p_{L\infty}$ is the pressure far from the bubble and σ is the liquid-vapor superficial tension. The mechanical evolution equation of the bubble (1) degenerates to the well known Rayleigh-Plesset equation when the interface temperature remains constant and equal to that of the liquid.

On the contrary, when the vapor bubble expansion produces significant interface cooling, equation (1) is no longer closed. An additional equation must be provided to calculate it. It is given by the solution of a convection diffusion equation with the following boundary conditions. The temperature at infinity is kept constant whereas the heat flux at the interface is deduced from the enthalpy balance at the interface. In addition, initial conditions must be provided. They are as follows,

- The pressure in the liquid is uniform and set to p_0.
- The temperature is uniform and set to T_0. Depending on particular circumstances, it may be larger or less than the saturation temperature corresponding to p_0.
- The initial value of the time derivative of the bubble radius is set to zero ($\dot{R}(0)=0$). The initial radius of the bubble, R_0, can be different from the unstable equilibrium radius given by,

$$R_{eq}=\frac{2\sigma}{p_v(T_0)-p_0}. \tag{2}$$

The evolution of the interface temperature has been solved by Plesset & Zwick (1952) and is based on the only assumption that the thermal boundary layer that develops beyond

σ N/m	ρ_L kg/m^3	h_{lv} MJ/kg	p_{v0} kPa	ρ_{v0} g/m^3	k_L W/m/K	D_L m^2/s
0.0724	997.8	2.448	2.65	19.4	0.602	1.44 10^{-7}

Table 1: Transport and thermodynamic properties of the liquid and vapor for the collapse test case.

the bubble interface is thin with respect to the bubble radius. The analytical solution to this problem is given by,

$$T(t) = T_0 - \left(\frac{D_L}{\pi}\right)^{\frac{1}{2}} \int_0^t \frac{R^2(x) \left.\frac{\partial T_L}{\partial r}\right|_{r=R(x)}}{\left[\int_x^t R^4(y)dy\right]^{1/2}} \, \mathrm{d}x, \tag{3}$$

where D_L is the thermal diffusivity of the liquid and the temperature gradient at the interface in the liquid phase is deduced from the enthalpy balance of the interface by assuming consistently no heat flux into the vapor,

$$\left.\frac{\partial T_L}{\partial r}\right)_{r=R(t)} = \frac{h_{lv}\rho_v(T)\dot{R}}{k_L}, \tag{4}$$

where h_{lv} is the heat of vaporization of the liquid, k_L is the heat conductivity of the liquid and $\rho_v(T)$ is the vapor density at the temperature of the interface for saturation conditions.

3 Bubble collapse: case 24-1 (PA)

The initial conditions of this problem are those proposed by Zwick & Plesset (1954) and are relative to previous experiments by Plesset. The liquid is initially subcooled so that the bubble shrinks continuously. The initial conditions are as follows,

$$p_0 = 0.544 \text{ atmosphere} \approx 0,544 \times 101.3 \text{ kPa}, \tag{5}$$

$$T_0 = 22 \text{ °C}, \tag{6}$$

$$R_0 = 2.5 \text{ mm}. \tag{7}$$

The transport and thermodynamic properties of the liquid and the vapor for the initial conditions are given in Table 1.

It is requested to calculate the variations of the bubble radius, the temperature of the interface and the velocity of the interface with time. The numerical solution of (1) and (3) is shown in figure (1). The bubble evolution is clearly similar to the Rayleigh regime since the heating of the interface only appears at the later stage of the collapse. This results from the very low saturation pressure of the vapor and is related to the high initial subcooling of the liquid. The numerical solution is provided as a text file and tables extracted from Lemonnier (2001). A selection of numerical results is also shown in table 2. The numerical solution in the Rayleigh regime is shown in table 3.

4 Initial stage of the growth of a vapor bubble, case 24-2 (PA)

The initial conditions for this test-case are from Zwick & Plesset (1954) and correspond roughly to the condition of the experimental test-case to be described later. The liquid

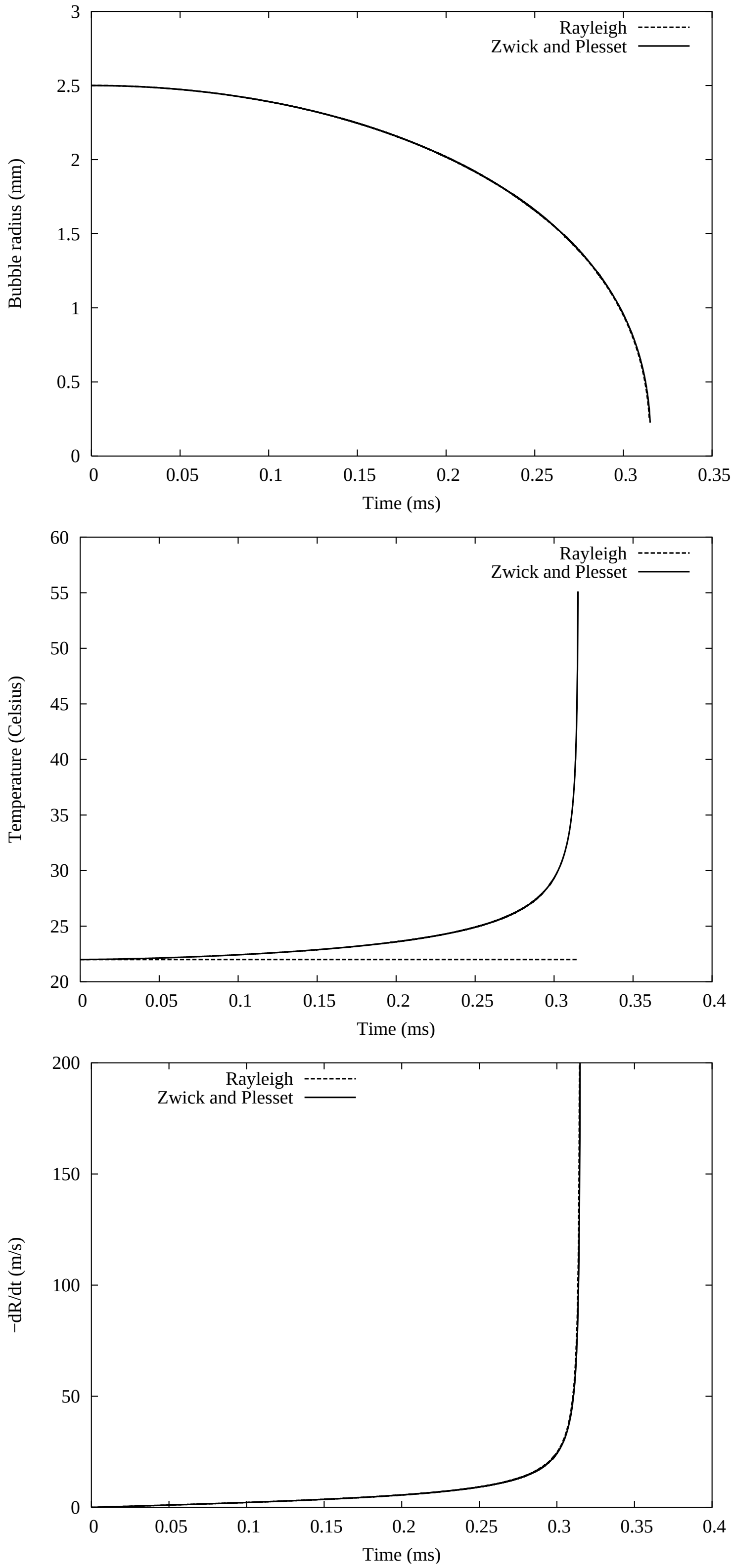

Figure 1: From top to bottom, time variations of the radius, the interface temperature and the negative of the interface velocity for a bubble of initial radius $R_0 = 2.5$ mm immersed in a liquid at an initial temperature of 22°C and a pressure of 0.544 atmosphere. Solution by the Zwick and Plesset algorithm and the Rayleigh model. Calculation: `Zwick et Plesset01.for`, plot: `zw01.plt`. Data files : `zw01.txt` and `zw01-R`.txt. Values at selected values of time are given in Table 2 and 3.

t_n (ms)	R_n (mm)	T_n (°C)	$\dot{R}_n$ (m/s)
.00000	2.5000	22.00	.000
.01001	2.4989	22.01	-.232
.02005	2.4958	22.03	-.434
.03015	2.4904	22.06	-.644
.04037	2.4828	22.09	-.863
.05047	2.4730	22.14	-1.081
.06048	2.4611	22.18	-1.303
.07058	2.4468	22.24	-1.532
.08066	2.4302	22.29	-1.766
.09068	2.4114	22.36	-2.004
.10074	2.3900	22.43	-2.254
.11082	2.3660	22.51	-2.512
.12085	2.3395	22.59	-2.781
.13087	2.3102	22.68	-3.063
.14092	2.2780	22.78	-3.360
.15092	2.2428	22.89	-3.674
.16095	2.2043	23.01	-4.011
.17099	2.1623	23.14	-4.374
.18100	2.1166	23.28	-4.766
.19100	2.0668	23.44	-5.195
.20101	2.0126	23.61	-5.667
.21102	1.9532	23.81	-6.198
.22105	1.8881	24.03	-6.803
.23109	1.8164	24.29	-7.502
.24111	1.7374	24.59	-8.323
.25113	1.6491	24.95	-9.323
.26116	1.5496	25.39	-10.581
.27118	1.4359	25.94	-12.231
.28119	1.3026	26.68	-14.572
.29122	1.1396	27.76	-18.284
.30126	.9247	29.65	-25.634
.31131	.5696	35.23	-54.615
.31510	.2250	55.11	-246.093

Table 2: Collapse of a steam bubble. $p_0 = 0.544$ atmosphere, $T_0 = 22$ °C, $R_0 = 2.5$ mm, Model of Zwick & Plesset (1954), data file: ZW01.txt.

t_n (ms)	R_n (mm)	T_n (°C)	$\dot{R}_n$ (m/s)
.00000	2.5000	22.00	.000
.01001	2.4989	22.00	-.232
.02005	2.4958	22.00	-.434
.03015	2.4904	22.00	-.644
.04037	2.4828	22.00	-.863
.05047	2.4730	22.00	-1.081
.06048	2.4611	22.00	-1.303
.07058	2.4468	22.00	-1.532
.08066	2.4302	22.00	-1.767
.09068	2.4114	22.00	-2.005
.10074	2.3899	22.00	-2.255
.11082	2.3659	22.00	-2.514
.12085	2.3394	22.00	-2.784
.13088	2.3101	22.00	-3.066
.14092	2.2778	22.00	-3.363
.15093	2.2427	22.00	-3.679
.16096	2.2041	22.00	-4.016
.17101	2.1620	22.00	-4.380
.18103	2.1161	22.00	-4.774
.19104	2.0662	22.00	-5.206
.20105	2.0118	22.00	-5.681
.21109	1.9522	22.00	-6.216
.22109	1.8870	22.00	-6.823
.23111	1.8153	22.00	-7.526
.24112	1.7360	22.00	-8.353
.25114	1.6474	22.00	-9.362
.26117	1.5474	22.00	-10.633
.27119	1.4330	22.00	-12.306
.28121	1.2987	22.00	-14.689
.29122	1.1346	22.00	-18.473
.30127	.9167	22.00	-26.082
.31128	.5524	22.00	-57.201
.31462	.2514	22.00	-201.231

Table 3: Collapse of a steam bubble in the Rayleigh regime (constant interface temperature). $p_0 = 0.544$ atmosphere, $T_0 = 22$ °C, $R_0 = 2.5$ mm, Model of Zwick & Plesset (1954), data file: ZW01-R.txt.

σ N/m	ρ_L kg/m^3	h_{lv} MJ/kg	p_{v0} MPa	ρ_{v0} kg/m^3	k_L W/m/K	D_L m^2/s
0.0583	956.2	2.248	0.1127	0.660	0.680	1.68 10^{-7}

Table 4: Transport and thermodynamic properties for the simulation of the initial stage of the growth of a vapor bubble.

is initially slightly superheated by a few Kelvin to remain consistent with the model assumptions. There exists in these conditions an unstable equilibrium radius satisfying both the thermodynamic and mechanical equilibrium conditions (2). These conditions are the following:

$$p_0 = 1 \text{ atmosphere} \approx 1.013 \text{ bar}, \tag{8}$$

$$T_0 = 103 \text{ °C}, \tag{9}$$

$$R_0 = (1+\epsilon)R_{eq} \approx 10.27 \ \mu\text{m}. \tag{10}$$

The thermodynamic and transport properties of the liquid and the vapor at the initial temperature and pressure are given in Table 4.

The initial conditions correspond to an unstable equilibrium state. The evolution of the system from these conditions is however "frozen" since they corresponds to a stationary point of (2) and (3). It is therefore necessary to initiate the instability either by heating up the liquid at small rate either by starting the calculation with a slightly larger radius than the equilibrium value.

Zwick & Plesset (1954) have shown that the growth proceeds along three phases. Each steps has been solved by an asymptotic analysis including the matching of them. This is a very cumbersome procedure and a numerical algorithm was proposed by these authors and was implemented by Lemonnier (2001). The three steps of the bubble growth are the following:

- A latent period, the details and length of which depends on the particular way to destabilize the bubble, such as the heating rate of the liquid or the relative excess, ϵ, of the initial radius with respect to the equilibrium radius (2).
- The initial growth, which can be described by a linearized version of (1) and (3).
- The fully developed growth solved by expanding the solution for long times.

Plesset & Zwick (1954) have shown that the time origin for the asymptotic solution for long times was arbitrary. Therefore, to compare the model solution to experiments, these authors shifted the asymptotic solution in time to get the best agreement with the data. This procedure would have been unnecessary if the full 3-zone solution would have been used. However, the solution depends critically on the initial process that triggers the instability.

Lemonnier (2001) shows that ϵ plays he same role as the initial heating of the liquid to start the bubble growth: for different values of ϵ, the radius evolves along parallel paths. To get the same latent period than Zwick & Plesset (1954) who used a heating rate of the liquid of 0.01°C/s, it is sufficient to select an initial radius slightly larger that

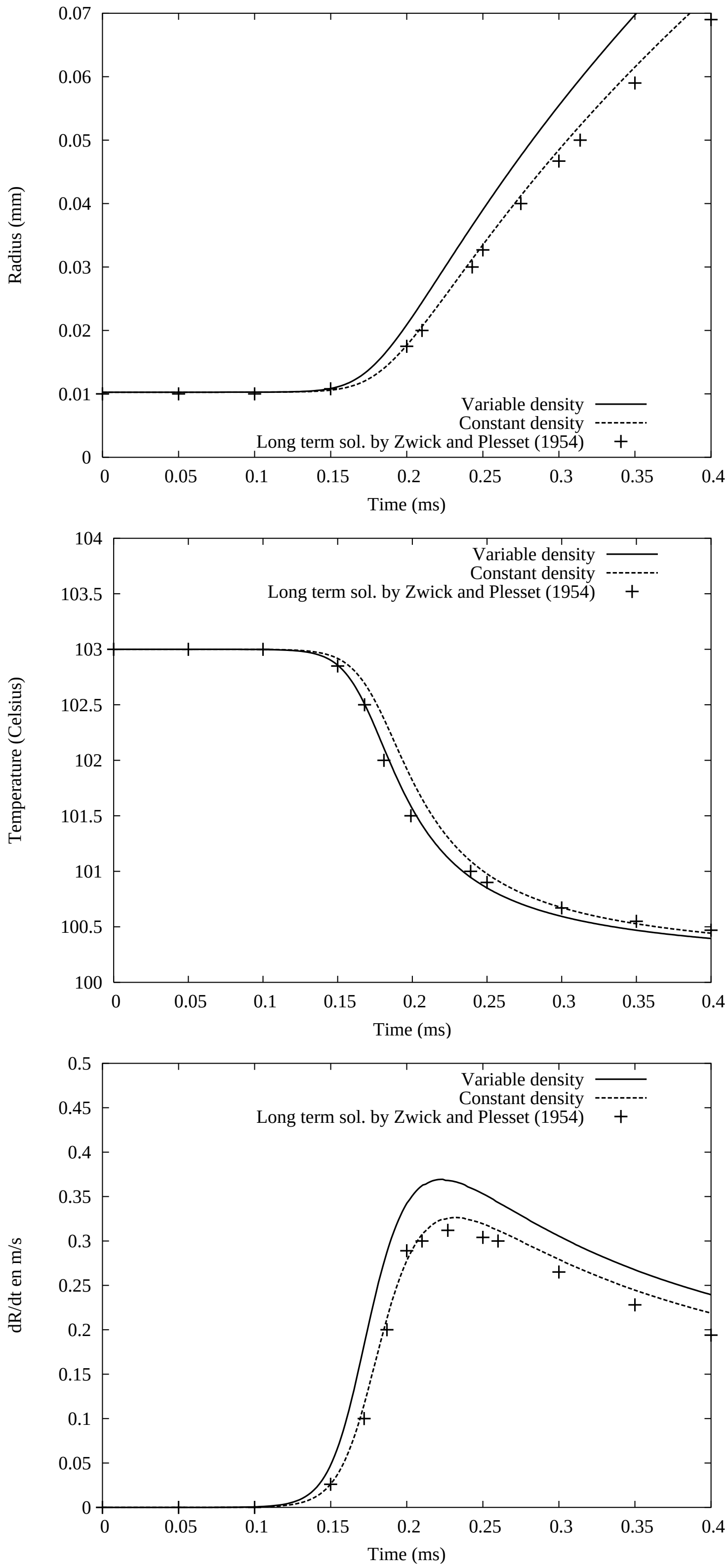

Figure 2: From top to bottom, time variations of the radius, the interface temperature and the interface velocity for a bubble of initial radius $R_0 = 10.27$ μm immersed in a liquid at an initial temperature of 103°C and a pressure of 1 atmosphere. Solution by the Zwick and Plesset algorithm with constant or variable vapor density. Symbols: sample points extracted from the asymptotic solution of Zwick & Plesset (1954). Calculation: `Zwick et Plesset02.for`, plot: `zw02.plt`. Data files : `zw02.txt` and `zw02r1`.txt, columns number 5,6,7 and 8. Files: `zwr-ref.txt`, `zwt-ref.txt` and `zwrd-ref.txt`. Values at selected times of the reference solution are given in Table 5 and 6.

t_n (ms)	R_n (mm)	T_n (°C)	$\dot{R}_n$ (m/s)
.00000	.0103	103.00	.000
.02600	.0103	103.00	.000
.05200	.0103	103.00	.000
.07800	.0103	103.00	.000
.10396	.0103	103.00	.001
.12957	.0104	102.97	.009
.15552	.0112	102.78	.072
.18140	.0153	102.09	.253
.20741	.0235	101.40	.358
.23275	.0328	101.01	.366
.25870	.0420	100.79	.345
.28508	.0508	100.65	.319
.31119	.0588	100.56	.296
.33643	.0660	100.50	.277
.36253	.0730	100.45	.260
.38799	.0795	100.41	.246
.41428	.0858	100.38	.233
.43933	.0915	100.35	.222
.46470	.0970	100.33	.213
.49050	.1024	100.32	.204
.51602	.1075	100.30	.197
.54211	.1125	100.29	.190
.56815	.1174	100.27	.184
.59467	.1222	100.26	.178
.62047	.1267	100.25	.173
.64556	.1310	100.24	.168
.67108	.1352	100.24	.164
.69766	.1395	100.23	.160
.72307	.1435	100.22	.156
.74904	.1475	100.22	.152
.77527	.1515	100.21	.149
.80030	.1552	100.20	.146
.82547	.1588	100.20	.143
.85064	.1624	100.20	.141
.87669	.1660	100.19	.138
.90242	.1695	100.19	.135
.92783	.1730	100.18	.133
.95290	.1763	100.18	.131
.97905	.1797	100.18	.129
1.00074	.1824	100.17	.127

Table 5: Growth of a steam bubble. $p_0 = 1$ atmosphere, $T_0 = 103$ °C, $R_0 = (1 + 5\ 10^{-8})R_{eq}$, Model of Zwick & Plesset (1954), data file: ZW02.txt.

t_n (ms)	R_n (mm)	T_n (°C)	$\dot{R}_n$ (m/s)
.00000	.0103	103.00	.000
.02600	.0103	103.00	.000
.05200	.0103	103.00	.000
.07800	.0103	103.00	.000
.10397	.0103	103.00	.001
.12973	.0103	102.98	.005
.15553	.0108	102.87	.040
.18151	.0134	102.36	.178
.20721	.0198	101.64	.301
.23229	.0278	101.18	.326
.25813	.0361	100.91	.313
.28418	.0440	100.74	.292
.30949	.0511	100.64	.272
.33512	.0578	100.56	.254
.36119	.0642	100.51	.238
.38675	.0701	100.46	.225
.41237	.0758	100.43	.213
.43761	.0810	100.40	.204
.46363	.0862	100.37	.195
.48872	.0910	100.35	.187
.51392	.0956	100.34	.180
.53967	.1002	100.32	.174
.56612	.1047	100.31	.168
.59133	.1088	100.29	.163
.61662	.1129	100.28	.158
.64269	.1170	100.27	.154
.66819	.1208	100.26	.150
.69387	.1246	100.26	.146
.71949	.1283	100.25	.142
.74490	.1319	100.24	.139
.77003	.1354	100.23	.136
.79581	.1388	100.23	.133
.82194	.1423	100.22	.131
.84740	.1456	100.22	.128
.87375	.1489	100.21	.126
.89931	.1521	100.21	.123
.92542	.1553	100.20	.121
.95073	.1583	100.20	.119
.97640	.1614	100.20	.117
1.00028	.1641	100.19	.116

Table 6: Growth of a steam bubble under the assumption of constant vapor density. $p_0 = 1$ atmosphere, $T_0 = 103$ °C, $R_0 = (1 + 5\ 10^{-8})R_{eq}$, Model of Zwick & Plesset (1954), data file: ZW02r1.txt.

the equilibrium radius by a relative amount (see equation 10) $\epsilon = 5\,10^{-8}$.

The results of the numerical solution of (2) and (3) for the above mentioned initial conditions are shown in Figure 2. In these figures, the symbols represents the asymptotic solution for long times by Zwick & Plesset (1954). The continuous line represents the numerical solution of (1) and (3) according to the original developments by Zwick & Plesset (1954, Appendix 2) while the dashed line represents the solution obtained with a constant vapor density to be consistent with the asymptotic approach of Zwick & Plesset (1954). The variable density assumption has obviously a direct impact on the onset of the instability.

It is requested to calculate the time variations of the bubble radius, interface temperature and velocity. Results are shown in Figure 2 and available as text files and arrays in Lemonnier (2001). Values at selected times of these solutions are given in Tables 5 and 6.

5 Thermally controlled growth of a vapor bubble (24-3)

This last proposed test-case corresponds to an experiment described by Plesset. The initial conditions are the known values of the pressure and the liquid temperature. However, the initial radius of the bubble is unknown in the experiment since it is hardly measurable. To analyze this situation, Plesset & Zwick (1954) have proposed to shift the real time origin such that the time evolution of the solution agrees with the observed results. In their asymptotic analysis, Plesset & Zwick (1954) have chosen the equilibrium radius as an initial condition whereas for solving the evolution equations (1) and (3) we have again chosen $\epsilon = 10^{-8}$. The latent period of the bubble growth is almost negligible with this parameter value with respect to the overall simulation time and there is therefore no need to take care of the physical time origin of the problem. The initial conditions of the calculation and the experiment are the following,

$$p_0 = 1 \text{ atmosphere} \approx 101.3 \text{ kPa}, \tag{11}$$

$$T_0 = 103.1^{\circ}\text{C}, \tag{12}$$

$$R_0 = (1+\epsilon)R_{eq} \approx 9.92\ \mu\text{m}. \tag{13}$$

The physical and transport properties of the liquid and the vapor are given in Table 7. Figure 3 shows that the model of Zwick & Plesset (1954) is in good agreement with the

σ N/m	ρ_L kg/m^3	h_{lv} MJ/kg	p_{v0} MPa	ρ_{v0} kg/m^3	k_L W/m/K	D_L m^2/s
0.0583	956.1	2.248	0.1131	0.620	0.680	$1.685\ 10^{-7}$

Table 7: Physical and transport properties of the liquid and the vapor for the growth of a vapor bubble.

data. The time evolution of the radius slightly differs depending on the assumption of constant or variable vapor density. However, it is clear that neglecting the cooling of the interface by the liquid evaporation induces a much faster growth of the bubble (the Rayleigh regime), which disagrees with the experiment.

It is requested to calculate the time evolution of the bubble radius, the interface temperature and its velocity. The numerical values of the to be used as a reference are given by Lemonnier (2001) and are provided at selected values of time in Tables 8, 9 and 10.

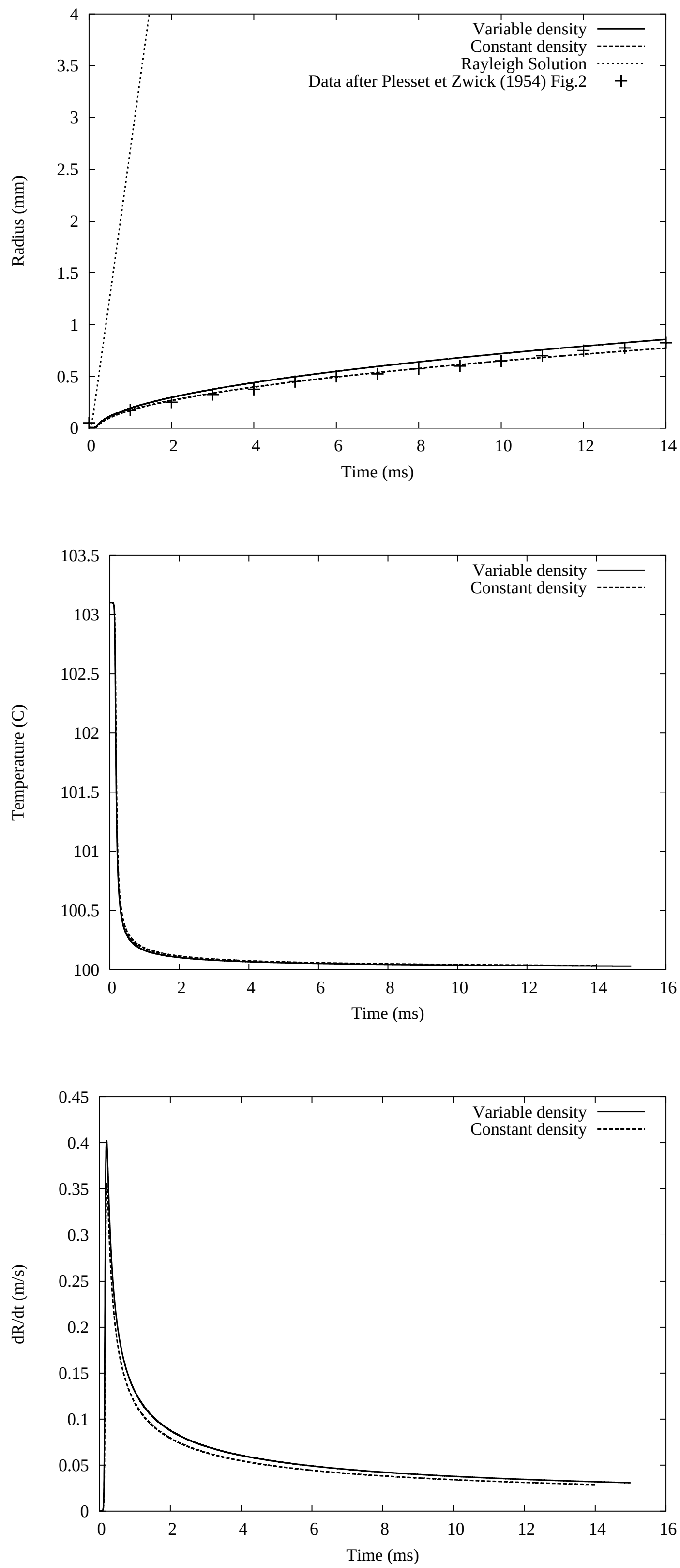

Figure 3: From top to bottom, growth of a vapor bubble with an initial radius equal to 9.92 μm immersed in water at 103.1°C and a pressure equal to 1 atmosphere. Solution after Zwick & Plesset (1954) considering the vapor density constant or variable with the temperature, Rayleigh solution (constant interface temperature), symbols: experimental data. Numerical model: Zwick et Plesset03.for, plot: zw03.plt. Numerical results : zw03fig2.txt, zwr1fig2.txt et zwrafig2.txt

t_n (ms)	R_n (mm)	T_n (°C)	$\dot{R}_n$ (m/s)
.00000	.0099	103.10	.000
.30110	.0643	100.51	.303
.60209	.1323	100.24	.179
.90238	.1792	100.18	.138
1.20323	.2173	100.14	.117
1.50384	.2502	100.12	.103
1.80442	.2795	100.11	.093
2.10485	.3062	100.10	.085
2.40522	.3309	100.09	.079
2.70547	.3540	100.09	.075
3.00604	.3757	100.08	.070
3.30702	.3964	100.08	.067
3.60770	.4161	100.07	.064
3.90834	.4349	100.07	.061
4.21002	.4531	100.07	.059
4.51154	.4705	100.06	.057
4.81227	.4874	100.06	.055
5.11253	.5036	100.06	.053
5.41363	.5195	100.06	.052
5.71432	.5348	100.05	.050
6.01475	.5497	100.05	.049
6.31501	.5643	100.05	.048
6.61516	.5785	100.05	.047
6.91524	.5923	100.05	.046
7.21528	.6059	100.05	.045
7.51530	.6191	100.05	.044
7.81532	.6321	100.04	.043
8.11534	.6449	100.04	.042
8.41536	.6574	100.04	.041
8.71540	.6696	100.04	.041
9.01619	.6817	100.04	.040
9.31690	.6936	100.04	.039
9.61819	.7053	100.04	.039
9.91933	.7169	100.04	.038
10.21976	.7282	100.04	.037
10.52016	.7393	100.04	.037
10.82103	.7503	100.04	.036
11.12277	.7612	100.04	.036
11.42387	.7719	100.04	.035
11.72486	.7825	100.03	.035
12.02533	.7929	100.03	.034
12.32653	.8032	100.03	.034
12.62797	.8134	100.03	.034
13.23010	.8334	100.03	.033
13.83057	.8528	100.03	.032
14.43201	.8719	100.03	.031
15.00001	.8895	100.03	.031

Table 8: Growth of a steam bubble. p_0 = 1 atmosphere, T_0 = 103.1 °C, R_0 = 9.92 μm, under the assumption of variable density.

t_n (ms)	R_n (mm)	T_n (°C)	$\dot{R}_n$ (m/s)
.00000	.0099	103.10	.000
.30085	.0564	100.58	.277
.60226	.1184	100.27	.162
.90229	.1608	100.20	.125
1.20296	.1952	100.16	.106
1.50316	.2249	100.14	.093
1.80327	.2514	100.12	.084
2.10374	.2755	100.11	.077
2.40438	.2979	100.10	.072
2.70455	.3187	100.10	.067
3.00521	.3384	100.09	.064
3.30600	.3570	100.08	.061
3.60670	.3748	100.08	.058
3.90810	.3919	100.08	.055
4.20903	.4082	100.07	.053
4.50954	.4239	100.07	.051
4.81022	.4391	100.07	.050
5.11045	.4538	100.06	.048
5.41070	.4681	100.06	.047
5.71202	.4820	100.06	.045
6.01231	.4955	100.06	.044
6.31271	.5086	100.06	.043
6.61283	.5214	100.06	.042
6.91345	.5339	100.05	.041
7.21505	.5462	100.05	.040
7.51546	.5582	100.05	.039
7.81546	.5699	100.05	.039
8.11565	.5814	100.05	.038
8.41641	.5928	100.05	.037
8.71683	.6039	100.05	.037
9.01736	.6148	100.05	.036
9.31833	.6255	100.05	.035
9.61900	.6360	100.04	.035
9.92066	.6465	100.04	.034
10.22168	.6567	100.04	.034
10.52326	.6668	100.04	.033
10.82478	.6767	100.04	.033
11.12577	.6865	100.04	.032
11.42584	.6962	100.04	.032
11.72601	.7057	100.04	.031
12.32737	.7244	100.04	.031
12.92777	.7426	100.04	.030
13.52972	.7604	100.04	.029
14.00044	.7740	100.04	.029

Table 9: Growth of a steam bubble. $p_0 = 1$ atmosphere, $T_0 = 103.1$ °C, $R_0 = 9.92$ μm, under the assumption of constant density.

t_n (ms)	R_n (mm)	T_n (°C)	$\dot{R}_n$ (m/s)
.00000	.0099	103.10	.000
.30060	.6901	75.31	2.830
.60140	1.5450	61.38	2.849
.90261	2.4040	51.01	2.854
1.20284	3.2613	42.40	2.857
1.50383	4.1213	34.84	2.858
1.80422	4.9800	28.06	2.859
2.10498	5.8399	21.82	2.860
2.40607	6.7010	16.03	2.860
2.70660	7.5605	10.61	2.860
3.00030	8.4006	5.60	2.861

Table 10: Growth of a steam bubble. $p_0 = 1$ atmosphere, $T_0 = 103.1$ °C, $R_0 = 9.92$ μm , under the assumption of constant interface temperature (Rayleigh regime).

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