Test-case number 28: The Lock-Exchange Flow

March, 17, 2003

Djamel Lakehal, Nuclear Engineering Laboratory, ETH Zurich
ETH-Zentrum, CLT, CH-8092 Zurich, Switzerland
Phone: +41 1 6324613; E-Mail: *lakehal@iet.mavt.ethz.ch*

Hervé Lemonnier, DER/SSTH/LIEX, CEA/Grenoble, 38054 Grenoble cedex 9, France
Phone: +33 (0)4 38 78 45 40, Fax: +33 (0)4 38 78 50 45, E-Mail: *herve.lemonnier@cea.fr*

1 Practical significance and interest of the test-case

The lock flows belong to the category of large-scale, gravity-driven currents, in which surface tension can be neglected. Gravity driven flows are induced by density variations due to a difference in temperature, such as atmospheric fronts, or due to the presence of a dispersed solid phase or a heavier dense gas. These are simple flow configurations, which may, however, result in very complex flows characterized by physical processes such as the emergence of Kelvin-Helmholtz-like instabilities, the formation of lobes and clefts at the front leading edge, *etc.* The lock flow consists of two fluids initially separated by a gate. Mutual penetration develops after the gate is withdrawn; a pair of gravity-driven fronts propagates along the upper and lower surfaces of the channel. A basic configuration of the flow is shown in figure 1. For density ratios of the order of one, the penetrations proceed almost symmetrically. However, the situation changes appreciably for higher density ratios, in that the lighter phase travels at much smaller speed than the dense gas underneath.

From a practical view point, the quantities of interest in these flows are (i) the front propagation velocity, U_f, and (ii) the run-out length, x_f. Both quantities depend primarily on the density ratio between the involved phases, ρ_g/ρ_a, where ρ_g designates the density of the dense gas, and ρ_a the density of the lighter gas, the dimensions of the channel as well as the effect of the wall boundary conditions, i.e. slip and non-slip, may also play an important role.

The interest of the test case is twofold: Evaluating the numerical scheme employed for the transport of the interface, and comparing the numerical results to the analytical solution derived on the basis of the Boussinesq fronts theory. The objective of the test case is to assess the capability of the scheme to conserve mass, and to deliver the right front shape and propagation velocity. Interface tracking schemes such as VOF and Level Sets could be employed.

The test case falls into the following categories:

- N: Purely numerical test designed to assess numerical schemes or part of them.
- P: Physical test-case where comparison to analytical solutions is required. It therefore belongs to the category:
    - PA: Comparison to a purely analytical solution.

The test case is attractive for its simplicity, and also because of the possibility to compare the results to the analytical solution.

2 Definitions and physical model description

The lock-exchange flow is considered for this computational exercise. The flow is accelerated during a short transient phase after the gate is withdrawn. The developing fronts attain a steady propagation velocity immediately after the transition phase. The simplest

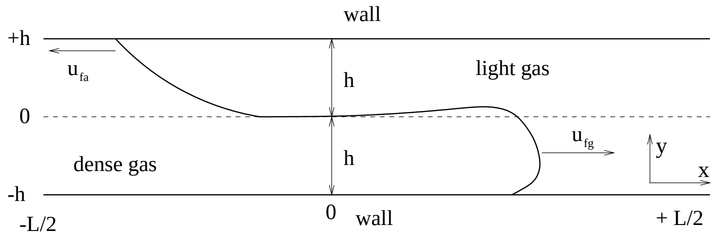

Figure 1: Basic sketch of the lock-exchange flow. The domain extends from $-L/2$ to $+L/2$ and from $-h$ to $+h$. The front velocities are denoted by U_{fg} and U_{fa}.

way to arrive at an analytical solution for the front propagation velocity is based on the following assumptions for the flow (Yih, 1965): (i) viscous dissipation is neglected, and (ii) the kinetic energy associated with the front motion is balanced by the loss by the system in potential energy. The scale of kinetic (ΔE_k) and potential energy (ΔE_p) is

$$\Delta E_k = \rho_m h U_f^3 \quad \text{and} \quad \Delta E_p = \frac{\Delta\rho g h^2 U_f}{2}, \tag{1}$$

respectively, where $\rho_m = (\rho_g + \rho_a)/2$ denotes the average density, $\Delta\rho = \rho_g - \rho_a$, and g is the acceleration of gravity. In the limit of small density ratios, the above hypothesis leads to the following relationship between the front speed and the buoyancy velocity, U_b:

$$U_f = U_{fa} = U_{fg} = \frac{1}{\sqrt{2}} U_b; \quad U_b = \sqrt{\frac{\Delta\rho}{\rho_m} hg} \tag{2}$$

3 Test-case description

It is proposed to calculate the evolution of the mutual intrusions at the lower boundary and the top boundary of the channel according to the parameters of table 1. Specifically, the front propagation velocity and run-out length will be displayed as a function of time, according to table 1. Note that the front speed U_f is defined as the speed at which the foremost point of the front travels, i.e. $U_f = dx_f/dt$, where x_f denotes the position of the nose in the longitudinal direction. In a second step, it is required to compare the front propagation velocities of both phases, U_{fg} versus U_{fa}, and draw a 3D map showing the onset of the front speed asymmetry (quantified by the ratio U_{fg}/U_{fa}) as a function of parameters R_1 and R_2 (c.f. table 1). This can be used to examine the validity of Yih's theory, which assumes slip wall-boundary conditions, flow velocities being uniform over the respective heights and equal to the front speed, and small density ratios.

The length scale characteristic of the problem is represented by the half channel-height denoted by h. The flow is two dimensional, incompressible, Newtonian, and laminar. The Navier-Stokes equations, without surface tension effects, should be resolved in time. The

Test-case	$R_1 = \rho_g/\rho_a$	$R_2 = \sqrt{\frac{\rho_g-\rho_a}{\rho_g+\rho_a}}$	$L/2h$
CO_2/Argon	1.11	0.22	5
CO_2/Argon	1.11	0.22	20
R22/Air	2.18	0.61	5
R22/Air	2.18	0.61	20
Argon/Helium	9.93	0.90	5
Argon/Helium	9.93	0.90	20
R22/Helium	21.6	0.95	5
R22/Helium	21.6	0.95	20
Water/Air	1000	0.999	5
Water/Air	1000	0.999	20

Table 1: Gas combinations used in the simulations of the lock flow.

computational domain consists of a channel of length L, *i.e.* $[-L/2, L/2] \times [2h]$. The computational grid suggested depends on the aspect ratio $L/2h$. Based on our earlier grid-sensitivity studies (Lakehal *et al.* , 2002), we suggest to use a grid consisting of 250×50 nodes, at least, for the $L/2h = 5$ case; the $L/2h = 20$ configuration requires 1000×50 computational nodes. The gate is initially located at $x/h = 0$, and the flow is at rest. The upper and lower boundaries should be treated using non-slip conditions (with friction). The vertical (end wall) planes have to be treated as frictionless walls. This treatment is suggested here in order to be consistent with the DNS of Hartel *et al.* (2000) of the same flow.

Summary of the required calculations

- The front propagation velocity (normalized by $\sqrt{2}U_b$) and run-out length (normalized by h) as a function of time.
- Compare the front propagation velocities of both phases, and establish a 3D map showing the onset of the front speed asymmetry as a function of parameters R_1 and R_2.
- Study the effect of the end walls on the front speeds as a function of their distance from the propagating front.

Results reported in Lakehal *et al.* (2002) obtained with the Level-Set method for the lock flow are shown in figure 2. The test case corresponds to the $L/2h = 5$ configuration. In Lakehal *et al.* (2002), attention was focused exclusively on the run-out length as a function of time. Figure 2 shows the interface evolution in the lock-exchange problem for $R_1 = 1.38$, where the front intrusions are clearly reproduced. The predicted run-out lengths of the dense and light gas for the CO_2/argon gas combination ($R_1 = 1.11$) have shown that the fronts have nearly equal velocities, in line with the Boussinesq theory of Yih. For density ratios higher than two (e.g. R22/argon), the dense-gas fronts were found to travel appreciably faster than the fronts of the light gas.

References

Hartel, C.J., Meiburg, E., & Necker, F. 2000. Analysis and Direct Numerical Simulation of the Flow at a Gravity-current Head. *Journal of Fluid Mechanics*, **418**(9), 189–212.

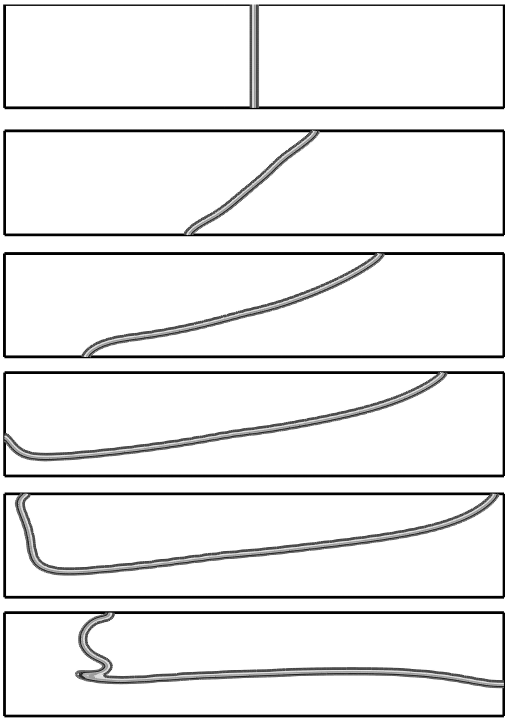

Figure 2: Interface evolution in the lock flow obtained by Level Sets for $R_1 = 1.38$; $R_2 = 0.4$; $L/2h = 5$. After Lakehal *et al.* (2002)

Lakehal, D., Meier, M., & Fulgosi, M. 2002. Interface tracking towards the direct simulation of heat and mass transfer in multiphase flows. *Journal of Heat and Fluid Flow*, **23**(3), 242–257.

Yih, C.S. 1965. *Dynamics of Nonhomogeneous Fluids*. Macmillan.